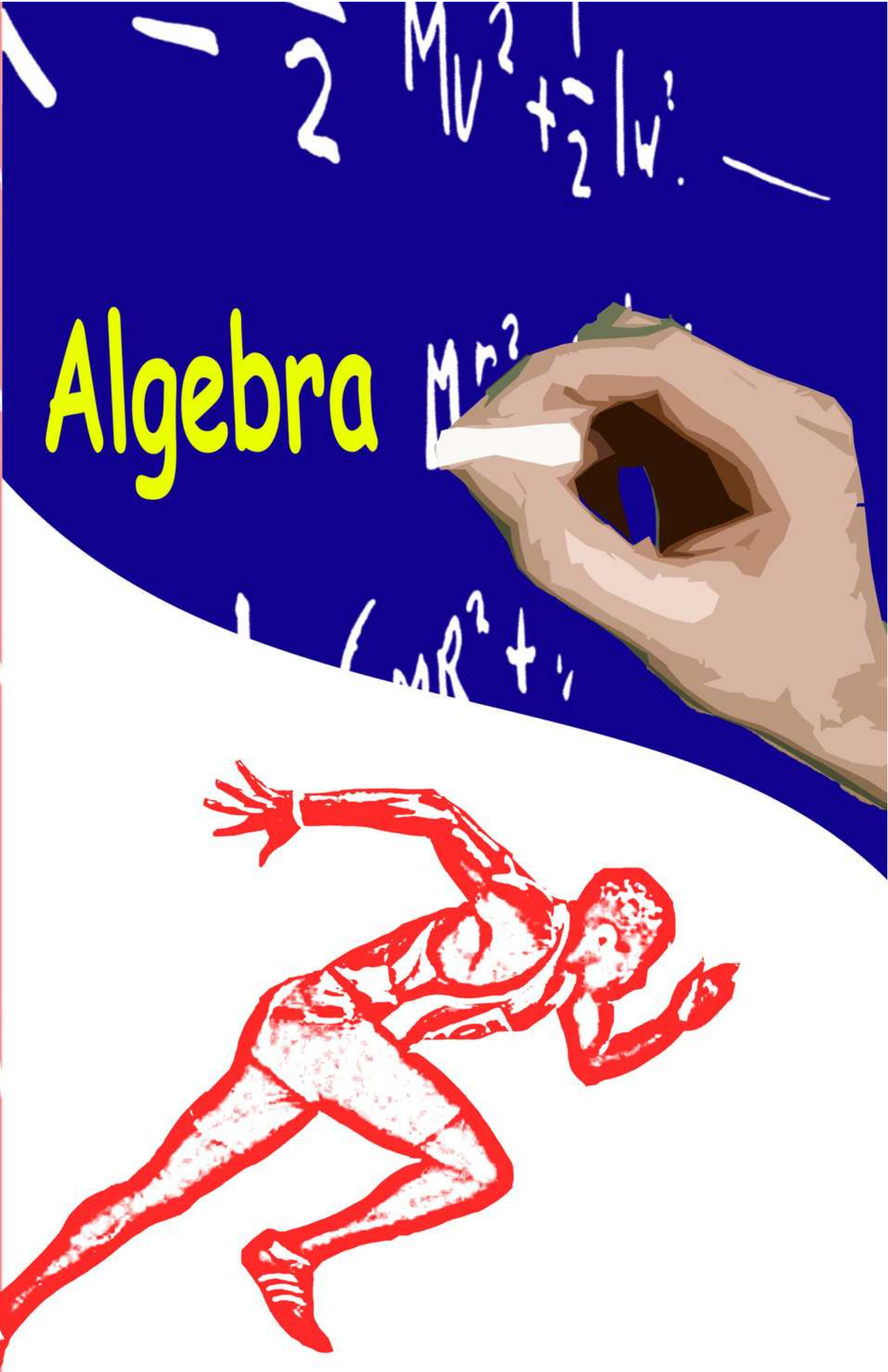
HEAD START MATHS

Algebra



CEMTL
Regional Centre for Excellence in Mathematics Teaching and Learning

centre for excellence
sigma Σ
in mathematics & statistics support

Foreword

The Regional Centre for Excellence in Mathematics Teaching and Learning (CEMTL) is collaboration between the Shannon Consortium Partners: University of Limerick, Institute of Technology, Limerick; Institute of Technology, Tralee and Mary Immaculate College of Education, Limerick., and is driven by the Mathematics Learning Centre (MLC) and The Centre for Advancement in Mathematics Education and Technology (CAMET) at the University of Limerick.

CEMTL is committed to providing high quality educational resources for both students and teachers of mathematics. To that end this package has been developed to a high standard and has been peer reviewed by faculty members from the University of Limericks Department of Mathematics and Statistics and sigma, the UK based Centre for Excellence in Teaching and Learning (CETL). Through its secondment programme, sigma provided funding towards the creation of these workbooks.

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3 Algebra

3.1 Introduction to Algebra

In mathematics we often have to deal with unknown numbers. Algebra is the process whereby we use letters to represent these unknown numbers. Let's look at an example:

Supposing when we add 5 to a certain number, we get 15.

What is the certain number? . . . Clearly the number is 10.

Using the letter x to represent this certain number, I can pose the above problem mathematically like this:

$x + 5 = 15$

so

$x = 10$

Let's take another example . . .

If I subtract 11 from a certain number, I get 4. Again let's use x to represent this unknown number.

$x - 11 = 4$.

The answer this time is $x = 15$.

So, as you can see, the value of x changes or varies depending on the problem or situation. For this reason we call x a **Variable**.

In the above examples, it was pretty easy for us to workout what value x held in each case. It is not always this straight forward so we need to be able to work with letters to solve various mathematical problems like we already do with numbers. There are many rules we adhere to when working with numbers. Similarly we have rules for working with expressions involving numbers and letters i.e. algebraic expressions. We are going to learn the basics in this workshop.

3.2 Multiplication of Letters and Numbers

When we multiply 2 numbers together we get another number, for example, $4 \times 5 = 20$. It is not the same for letters, for example, $a \times b$ = ??? For this reason we write $a \times b = ab$. We simply drop the multiplication sign and write the letters together.
Similarly $3 \times x = 3x$.

Example:

$6 \times x \times y = 6xy$

$5a \times 4b = 20ab$

$6x \times 2y \times 3z = 36xyz$

$7 \times p \times 3 \times q = 21pq$

We multiply the numbers as usual and then the letters together. We generally write the letters in alphabetical order.

Example:

$2 \times b \times c \times 2 \times d = 4bcd$ (which is the same as $4dcb$ or $4bdc$ etc.).

Some New Vocabulary

Coefficients

In the expression $4x + 7y$ we say that 4 is the x **Coefficient** (because x is multiplied by 4) and 7 is the y **Coefficient**.

Constants

In the expression $3x + 6$

6 is called a **Constant** because it is not multiplied by any variable.

Exercises 1

Multiply the Following

1. $4 \times s$

2. $7 \times x \times r$

3. -10 $\times\ q \times 2$

4. $a \times b \times c \times d$

5. $5c \times 4d$

6. $6xy \times -12ab$

7. $8ac \times 9bd$

8. -5$lm \times 2 \times -3w$

9. $10 \times -s \times 4 \times t$

10. $7abf \times 3eg$

3.3 Addition of Algebraic Terms

Terms which have precisely the same variables or letters are called **Like** terms.

For example, $3a$ and $4a$ are like terms whereas $5b$ and $6c$ are not like terms. Also $3a$ and $2ab$ are not like terms.

When adding in algebra we only combine like terms together.

If you add:

3 apples, 2 bananas, 1 apple and 4 bananas

What do you have?

4 apples and 6 bananas!

If we let a represent apples and b represent bananas we can write the above sum mathematically as:

$3a + 2b + 1a + 4b$

= $4a + 6b$

Exercises 2

Simplify the Following

1. $a + 4a - 10a$

2. $8x + 5y + 12x - y$

3. $8a + 3b + 2a + 12b$

4. $5c + 6d - 3c - 4d$

5. $2mn + 13 + 7mn - 10$

6. $11def + 6de + def - 2defg$

7. $5m + 12n - 4q + 2q - 11m$

8. -7 $- 8fg - 24 - 7fg$

9. $20jk + 16 - 20jk - 16$

10. $2ab + z + z - 3z - 4ab$

Exercises 3

Evaluate Each of the Following Terms by Substituting in the Appropriate Values for a, b and c, Given That a = 2, b = -1 and c = 3

1. $7b$

2. $a + b + c$

3. -$a - b - c$

4. $5a + 2b$

5. $4c - 2b$

6. $2bc - 6ac$

7. -15$a - bc$

8. $\dfrac{3a + 4b}{c}$

9. $\dfrac{-2c - a}{ab}$

10. $\dfrac{6ab - 10cb}{abc}$

3.4 Real Life Problems

In this section we will use the algebraic skills we have just learned to solve some real-world problems.

Problem 1

$C = \frac{5}{9}(F - 32)$

is the formula used to convert degrees Fahrenheit (F) into degrees Celsius (C).

What is the equivalent temperature in degrees Celsius when it is 64 degrees Fahrenheit?

Solution

F = 64

C = ?

$C = \frac{5}{9}(F - 32)$

Replace F with 64

$\Rightarrow C = \frac{5}{9}(64 - 32)$

$\Rightarrow C = \frac{5}{9}(32)$

$\Rightarrow C = 17.77$ degrees

Find the corresponding temperature in degrees Celsius if the temperature is 22 degrees Fahrenheit.

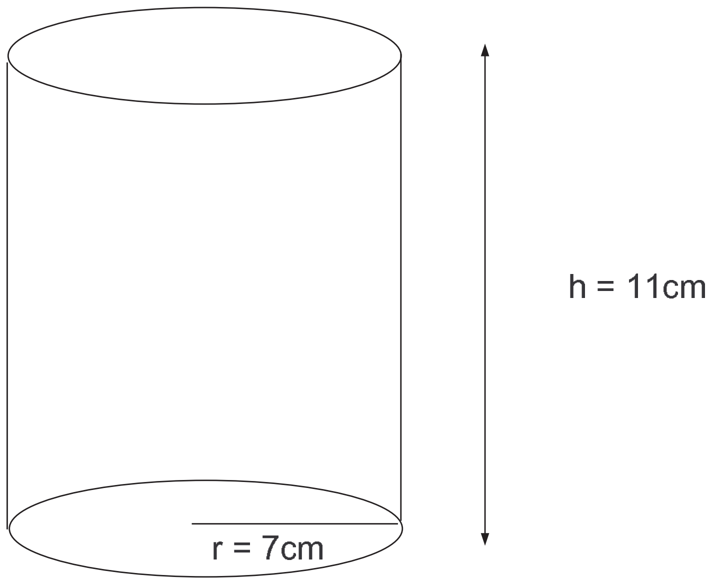
Problem 2

The volume of a cylinder (V)is given by the algebraic expression:

$V = \pi r^2 h$

where π is estimated to be 3.14, r is the radius of the cylinder and h is the height.

Find the volume of the cylinder below:



Solution

$\pi = 3.14cm; r = 7cm; h = 11cm$

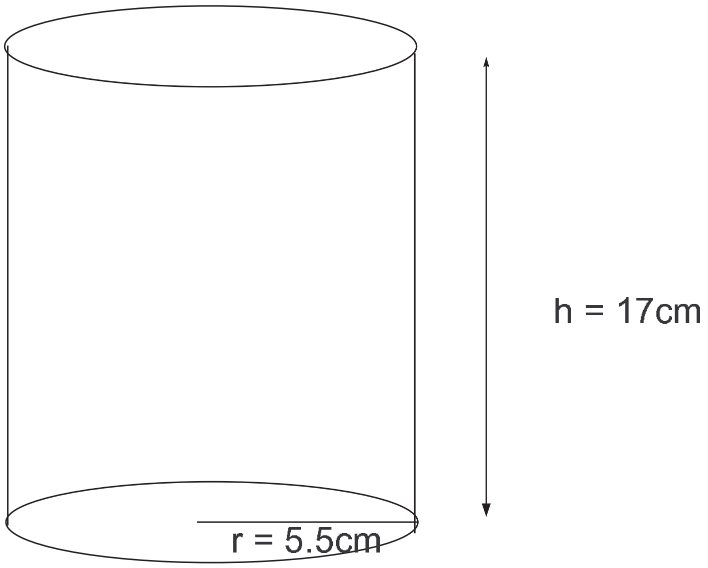
$V = \pi r^2 h$

$V = (3.14)(7)^2(11)$

$V = 1692.46cm^3$

Note: Area is two dimensional so it is measured in units^2. Volume is three dimensional so it is measured in units^3.

Find the volume of the cylinder below



Problem 3

One litre of petrol costs €1.29 and one can of oil costs €2.35.

Write down an expression for the cost of x litres of petrol and y cans of oil.

Determine the cost if a person buys 20 litres of petrol and 2 cans of oil.

Solution

Cost = $1.29x + 2.35y$

We know that

$x = 20$ and $y = 2$

We substitute the values for x and y into our formula:

Cost = $1.29(20) + 2.35(2)$

Cost = $25.80 + 4.70$

Cost = €30.50

At the same prices, determine the cost if a person buys 37 litres of petrol and 5 cans of oil.

Problem 4

To park in a new multistorey car park in the city, you pay a fixed charge of €2.00 and an additional 85 cent per hour or part thereof (i.e. if you park for an hour and 15 minutes, you pay for two hours). If C is used to represent the total cost of parking in this car park and t is the time in hours, write an expression to represent the relationship between the total cost of parking and t.

Use the expression derived to calculate the cost of parking your car in this carpark for 6 hours.

Solution

Cost = Fixed Charge + (Rate per hour $\times$ Number of hours car is parked)

$C = 2.00 + 0.85t$

We know that t = 6.

We substitute the value $t = 6$ into our expression:

$\Rightarrow C = 2.00 + 0.85(6)$

$\Rightarrow C = 2.00 + 5.10$

$\Rightarrow C$ = €7.10

How much would it cost you to park your car in this car park for 11 and a half hours

3.5 Multiplying out Brackets of the Form a(b + c)

When a number is written outside (immediately to the left of) a set of brackets it means that the number is to be multiplied by everything inside the brackets.

For example, $6(1+3)$ implies that 6 is multiplied by *both* 1 and 3.

$$\begin{aligned} & 6(1+3) \\ = \; & (6\times 1)+(6\times 3) \\ = \; & 6+18 \\ = \; & 24 \end{aligned}$$

Note that with numbers we would normally work out the brackets first to get $6\times 4 = 24$.

Example 1

In this example 5 is outside the brackets therefore 5 must be multiplied by both x and 2, one at a time.

$$\begin{aligned} & 5(x+2) \\ = \; & (5\times x)+(5\times 2) \\ = \; & 5x+10 \end{aligned}$$

Example 2:

$$\begin{aligned} & 7(3x-4) \\ = \; & (7\times 3x)-(7\times 4) \\ = \; & 21x-28 \end{aligned}$$

Example 3:

In Examples 3 and 4 we have 2 sets of multiplying to do. We simply do what we did in Examples 1 and 2 above, one set of brackets at a time, then finish off by adding like/similar terms to simplify.

$$\begin{aligned} & 2(4x-3)-3(x+5) \\ = & (2\times 4x-2\times 33)-(3\times x+3\times 5) \\ = & (8x-6)-(3x+15) \\ = & 8x-6-3x-15 \\ = & 5x-21 \end{aligned}$$

Example 4:

$$\begin{aligned} & 5(3a+2b)-(4a+3b) \\ = & (5\times 3a+5\times 2b)-(4a+3b) \\ = & (15a+10b)-(4a+3b) \\ = & 15a+10b-4a-3b \\ = & 11a+7b \end{aligned}$$

Exercises 4

Remove the Brackets and Simplify Where Possible

1. $15(f + 3g)$

2. $-2(d + 2b)$

3. $8(2l + 3m - n)$

4. -5$(-\alpha - 3\beta)$

5. -2$(2g - 4h - 7i)$

6. $10(30x - 20y - z)$

7. $10(7\delta - 2\theta)$

8. $3(x + 1) + 6(x + 4)$

9. $6(5\pi + 3) + 5(\pi - 4)$

10. - $4(2a + 5) - 3(a - 2)$

11. $7(\phi - 2) + 5(3\phi - 8)$

12. $2a(b + c) + 3a(2b + 7c)$

13. $7b(-a + c) + b(4a - 3c)$

14. $a(2b + c) - 5(ab - ac)$

15. $6x(y + z) - 2x(y + 2z)$

3.6 Indices

In our first workshop on Number Systems we learned how

$$2 \times 2 \times 2 = 2^3 \quad \text{and} \quad 2^4 \times 2 \times 2^2 = 2^7$$

We add the indices/powers when we multiply numbers with the same base.

Therefore

$$a \times a = a^2$$

$$a \times a \times a = a^3$$

$$x^2 \times x^4 = x^6$$

$$3y \times 2y^5 = 6y^6$$

$$5ab \times 3ab = 15a^2b^2$$

$$12x \times 2xy = 24x^2y$$

Exercises 5

Write Each the Following Without the Multiplication Signs

1. $z \times z \times z$

2. $\pi \times \pi \times \pi \times \pi$

3. $\alpha^2 \times \alpha^2$

4. $h^7 \times h$

5. $-2a \times -2a$

6. $-3y \times 4y^2$

7. $qr \times qs$

8. $tu \times ut$

9. $8\theta^3 \times -8\theta$

10. $\alpha(\alpha - 12)$

11. $10x(30x - 20)$

12. $4\sigma(5\sigma - 7)$

13. $\beta^2(\beta - 9)$

14. $x^3(-5x + 11)$

15. $-2a^2(7a - 5)$

Exercises 6

Evaluate Each of the Following When $a = 1$, $b = -2$ and $c = 3$

1. $a^3 - b^2$

2. $4ac + 3b^2$

3. $a^2 - b^2 + c^2$

4. $5a^2 - c^4$

5. $9b^4 - c^3$

6. $a^2bc - ab^2c$

7. $ab^2 - bc^2 + 4$

8. $10a^2bc$

9. $-13ab^2c$

10. $11a^2 - 11c^2 - 11$

Exercises 7

Remove the Brackets and Simplify

1. $2x(x-3)+4x(x+9)$

2. $3\mu(2\mu-8)+6\mu(\mu-1)$

3. $2\gamma(\gamma+16)-3\gamma(2\gamma-17)$

4. $11a(-4a-10)-3a(a+21)$

5. $6x(x^3+5x^2)+4x^2(x^2-3x)$

3.7 Multiplying out Brackets of the Form (a + b)(c + d)

In this section, we will demonstrate how to multiply out pairs of brackets involving algebraic expressions.

Take $(x + 5)(x + 3)$

The two sets of brackets above indicate that x and 5, the elements in the first brackets, must be multiplied by both x and 3, the elements in the second brackets. We can only multiply two numbers together at a time so we use the following method:

Break up the first brackets so that x is multiplied by $x + 3$ and then 5 is multiplied by $x + 3$

$x(x + 3) + 5(x + 3)$

We proceed as before

$(x)(x) + (x)(3) + (5)(x) + (5)(3)$

$x^2 \underline{+3x + 5x} + 15$

Add similar/like terms.

Answer: $x^2 + 8x + 15$

Exercises 8

Expand Each of the Following

1. $(x + 4)(x + 5)$

2. $(a + 10)(a + 10)$

3. $(x - 9)(x - 9)$

4. $(2c-1)(5c+2)$

5. $(5\epsilon-1)(\epsilon-3)$

6. $(2x-11)(x-11)$

7. $(6\tau-1)^2$

8. $(l + m)(p + q)$

9. $(7g + 3h)^2$

10. $(\phi - 1)(\phi^3 - \phi + 7)$

3.8 Answers

Exercise 1:

1). $4s$	2). $47xr$	3). $-20q$	4). $abcd$
5). $20cd$	6). $-72abxy$	7). $72abcd$	8). $30lmw$
9). $-4st$	10). $21abefg$		

Exercise 2:

1). $-5a$	2). $20x + 4y$	3). $10a + 15b$
4). $2c + 2d$	5). $9mn + 3$	6). $12def + 6de - 2defg$
7). $-6m + 12n - 2q$	8). $-31 - 15fg$	9). 0
10). $-2ab - z$		

Exercise 3:

1). -7	2). 4	3). -4	4). 8
5). 14	6). -42	7). -27	8). $\frac{2}{3}$
9). 4	10). -3		

Problem 1: $C = -5.6$ degrees

Problem 2: $V = 1614.75cm^3$

Problem 3: €59.48

Problem 4: €12.20

Exercise 4:

1). $15f + 43g$	2). $-2d - 4b$	3). $16l + 24m - 8n$
4). $5\alpha + 15\beta$	5). $-4g + 8h + 14i$	6). $300x - 200y - 10z$
7). $70\delta - 20\theta$	8). $9x + 27$	9). $35\pi\pi - 2$
10). $-11a - 14$	11). $22\phi - 54$	12). $8ab + 23ac$
13). $-3ab + 4bc$	14). $-3ab + 6ac$	15). $4xy + 2xz$

Exercise 5:

1). z^3	2). π^4	3). α^4
4). h^8	5). $4a^2$	6). $-12y^3$
7). q^2rs	8). u^2t^2	9). $-64\theta^4$
10). $\alpha^2 - 12\alpha$	11). $300x^2 - 200x$	12). $20\sigma^2 - 28\sigma$
13). $\beta^3 - 9\beta^2$	114). $-5x^4 + 11x^3$	15). $-14a^3 + 10a^2$

Exercise 6:

1). -3	2). 24	3). 6	4). -76
5). 117	6). -18	7). 26	8). -60
9). -156	10). -99		

Exercise 7:

1). $6x^2 + 30x$ 2). $12\mu^2 - 30\mu$ 3). $-4\gamma^2 + 83\gamma$ 4). $-47a^2 - 173a$

5). $10x^4 + 18x^3$

Exercise 8:

1). $x^2 + 9x + 20$ 2). $a^2 + 20a + 100$ 3). $x^2 - 18x + 81$
4). $10c^2 - c - 2$ 5). $5\epsilon^2 - 16\epsilon + 3$ 6). $2x^2 - 33x + 121$
7). $36\tau^2 - 12\tau + 1$ 8). $lp + lq + mp + mq$ 9). $49g^2 + 42gh + 9h^2$
10). $\phi^4 - \phi^3 - \phi^2 + 8\phi - 7$

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