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# Factorising





## Foreword

The Regional Centre for Excellence in Mathematics Teaching and Learning (CEMTL) is collaboration between the Shannon Consortium Partners: University of Limerick, Institute of Technology, Limerick; Institute of Technology, Tralee and Mary Immaculate College of Education, Limerick., and is driven by the Mathematics Learning Centre (MLC) and The Centre for Advancement in Mathematics Education and Technology (CAMET) at the University of Limerick.

CEMTL is committed to providing high quality educational resources for both students and teachers of mathematics. To that end this package has been developed to a high standard and has been peer reviewed by faculty members from the University of Limericks Department of Mathematics and Statistics and sigma, the UK based Centre for Excellence in Teaching and Learning (CETL). Through its secondment programme, sigma provided funding towards the creation of these workbooks.

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# 5 Factorising

## 5.1 Factors

In our first workshop on Number Systems we learned about factors.

For example:  $1 \times 6 = 6$  and  $2 \times 3 = 6$

Therefore 1, 2, 3, and 6 are all factors of 6.

In other words, all the numbers that divide exactly into 6.

In our Algebra workshop we learned how to multiply out brackets

We saw that  $5(a + b) = 5a + 5b$

Therefore 5 and  $(a + b)$  are factors of  $5a + 5b$ .

We also saw how  $(x + 5)(x + 3) = x^2 + 8x + 15$

Therefore  $(x + 5)$  and  $(x + 3)$  are factors of  $x^2 + 8x + 15$ .

And so on...

When we find the factors of an algebraic expression (called a **polynomial**) we say that the expression has been **factorised**.

In this workshop we will learn how to factorise many different types of polynomial.

## 5.2 Taking out a Common Factor

### Example 1

Factorise  $100d + 150e + 200f$

We can see that 50 divides into all 3 terms. So we take 50 outside a bracket and divide every term by 50

$$50 \left( \frac{100d}{50} + \frac{150e}{50} + \frac{200f}{50} \right)$$

$$50(2d + 3e + 4f)$$

### Example 2

Factorise  $4a + 24b - 16c$

We can see that 4 divides into all 3 terms.

$$4 \left( \frac{4a}{4} + \frac{24b}{4} - \frac{16c}{4} \right)$$

$$4(a + 6b - 4c)$$

### Example 3

Factorise  $3\beta^3 - 6\beta^2 + 9\beta$

Here we can divide all terms by  $3\beta$  i.e. take  $3\beta$  outside the bracket

$$3\beta \left( \frac{3\beta^3}{3\beta} - \frac{6\beta^2}{3\beta} + \frac{9\beta}{3\beta} \right)$$

$$3\beta(\beta^2 - 2\beta + 3)$$

### Example 4

Factorise  $12s^2t + 8st^2 - 6s^2t^2$

Here we can divide all terms by  $2st$

$$2st \left( \frac{12s^2t}{2st} + \frac{8st^2}{2st} - \frac{6s^2t^2}{2st} \right)$$

$$2st(6s + 4t - 3st)$$

## Exercises 1

Factorise Each of the Following

1.  $6s + 30t$

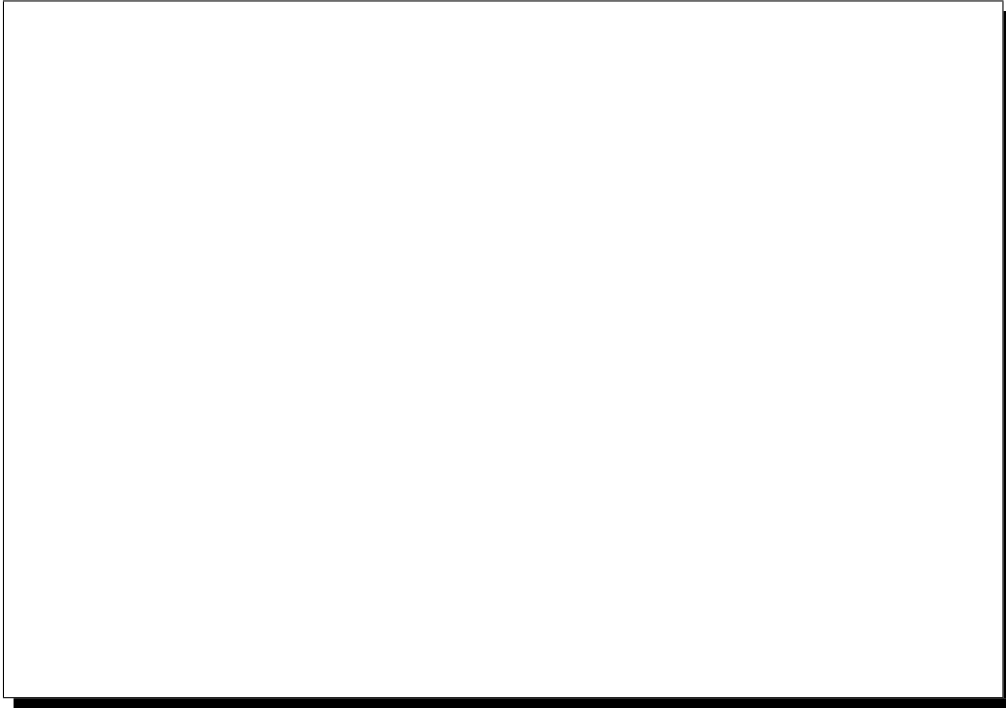


2.  $21u - 42v$

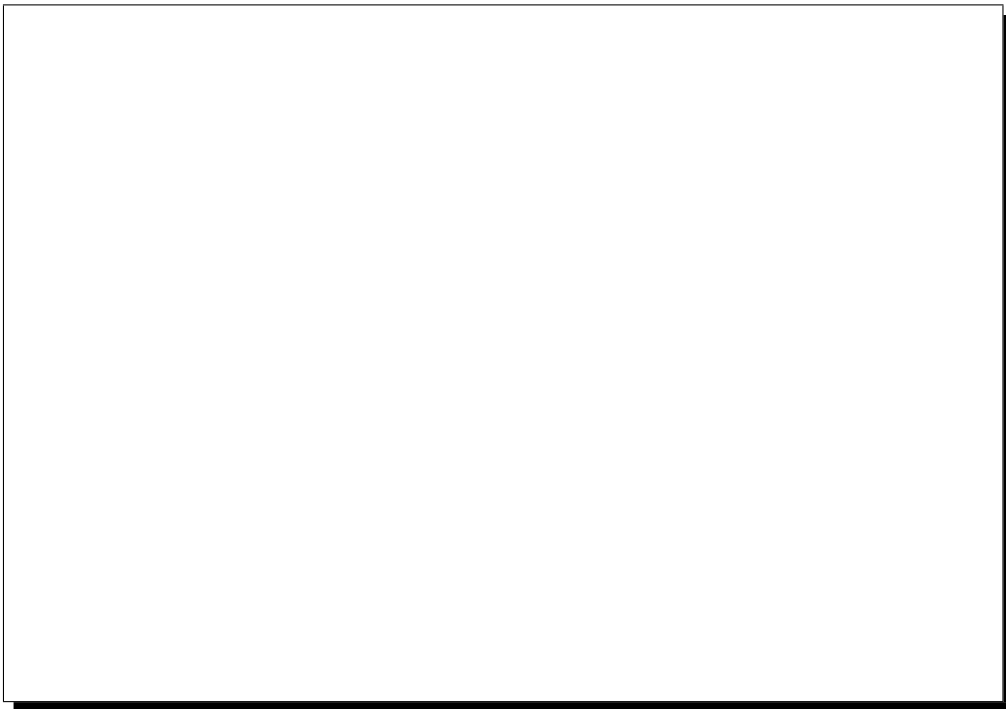


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3.  $-ab + ac$



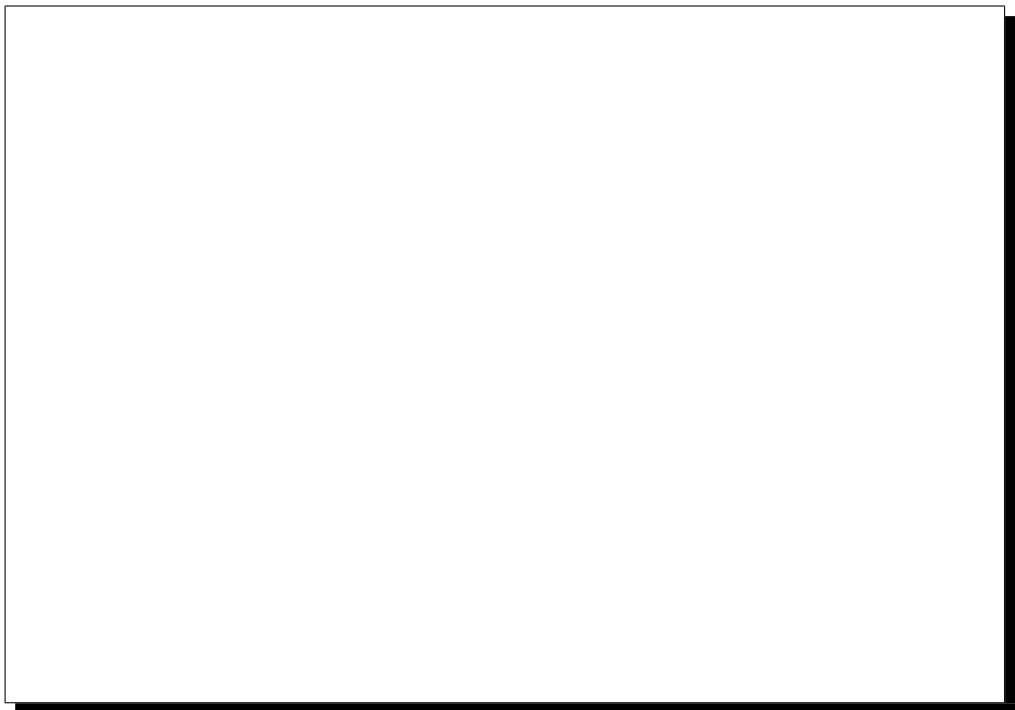
4.  $25\alpha\beta - 35\beta\delta$



5.  $21\theta^3 - 24\theta$



6.  $33q^4 - 22q^2$



7.  $-25\alpha\beta^2\gamma + 50\alpha^2\beta^2\gamma^2$



8.  $-4x^5 - 16x^4z - 8xy^3$



9.  $abc + a^3b^4c^2 - a^6b^{10}c^2$



10.  $l^2mn^2 - m^2n + ln^2$



## 5.3 Factorising by Grouping

Look at the expression

$$ax + ay + 6x + 6y$$

It is clear that there is no factor common to all four terms.

However if we look closely we see that  $a$  is common to the first 2 terms and 6 is common to the second 2 terms.

When this occurs we use a method called **Factorising by Grouping** to factorise such expressions.

What this means is we group terms together in pairs where there is a common factor and then factorise like before.

In this particular example, the first 2 terms have  $a$  in common and the second 2 terms have 6 in common.

$$(ax + ay) + (6x + 6y)$$

$$= a(x + y) + 6(x + y)$$

As you can see the terms inside both brackets are the same so ...

We can write the expression as  $(a + 6)(x + y)$

Therefore our factors are  $(a + 6)$  and  $(x + y)$ .

### Example

Factorise  $4ab + 4ac + 5xb + 5xc$

$$(4ab + 4ac) + (5xb + 5xc)$$

$$= 4a(b + c) + 5x(b + c)$$

$$= (4a + 5x)(b + c)$$

The factors are  $(4a + 5x)$  and  $(b + c)$

## Exercises 2

### Factorise the Following

1.  $qx + qy + 4x + 4y$



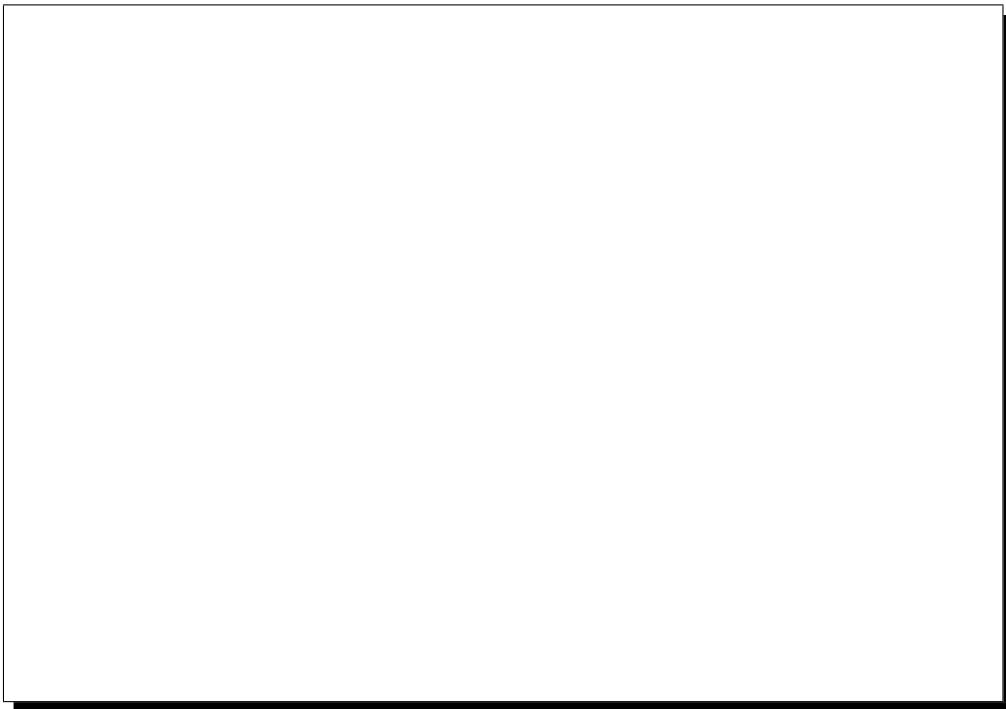
2.  $2bc - 2bd + 17c - 17d$



3.  $\pi\rho + \kappa\rho + \pi\lambda + \kappa\lambda$



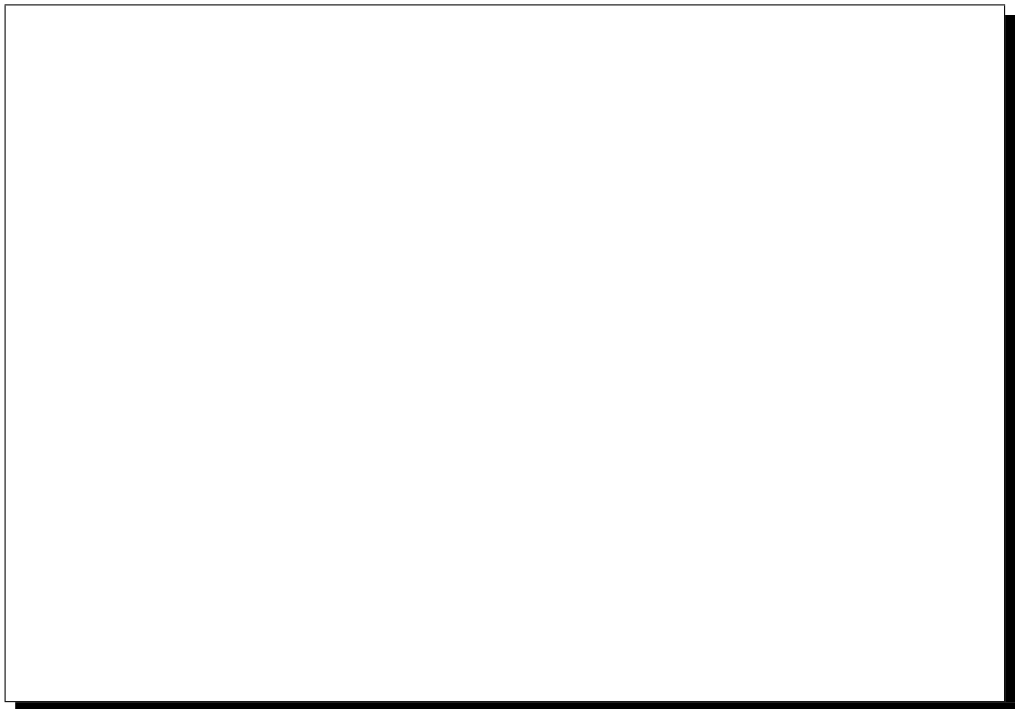
4.  $5as - 10bs + 2aq - 4bq$



5.  $tp - 2t + 16p - 32$



6.  $2l\alpha + \alpha\delta - 2l\phi - \delta\phi$

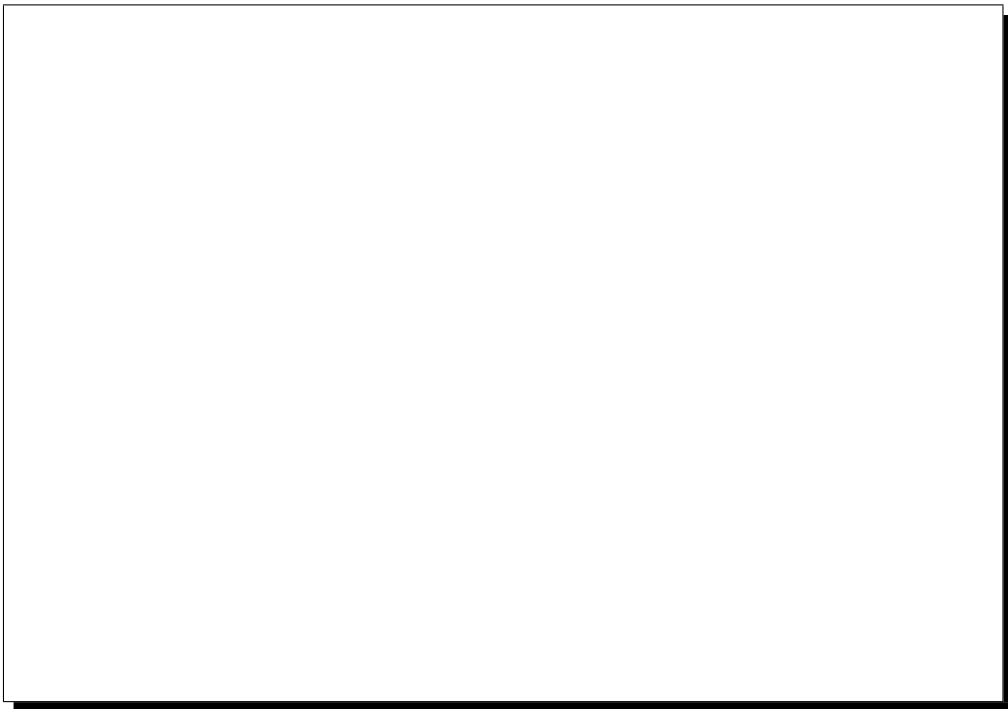


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7.  $uv - 2cd - vc + 2ud$



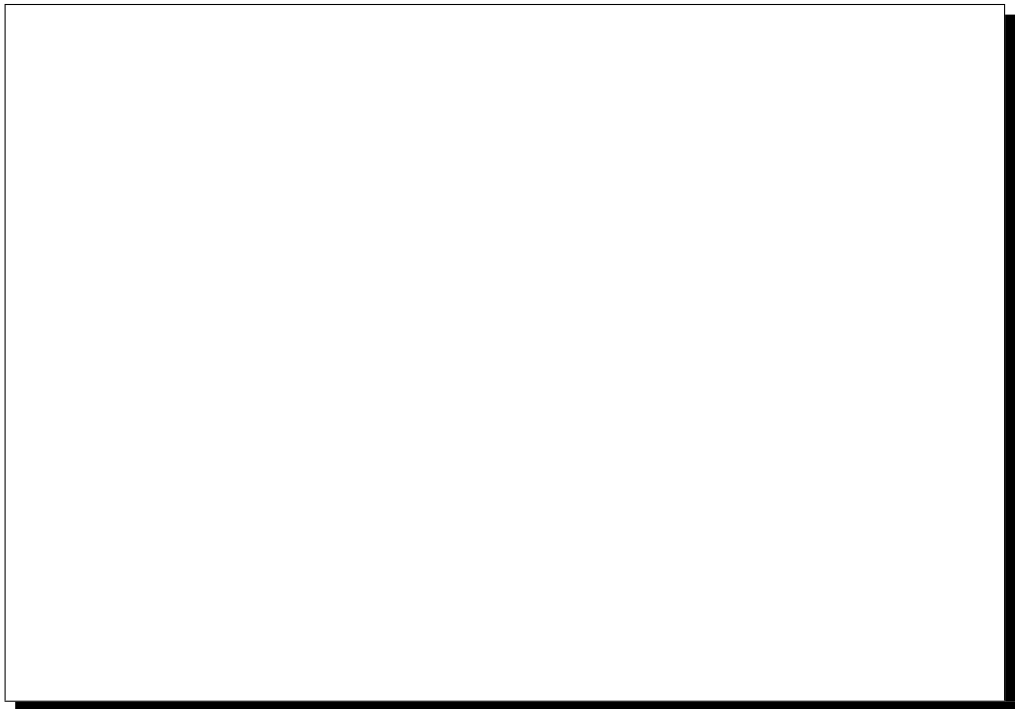
8.  $2\mu - 3\lambda\kappa - 6\lambda + \kappa\mu$



9.  $ih - 4ij + 8gj - 2gh$



10.  $16s + 2r - rs - 32$



## 5.4 Factorising Quadratics

A mathematical expression of the form

$ax^2 + bx + c$  (where  $a, b, c$  are constants) is called a **Quadratic** expression.

If we multiply out the following:

$$(a + 2)(a + 7)$$

We get

$$a(a + 7) + 2(a + 7)$$

$$= a^2 + 7a + 2a + 14$$

$$= a^2 + 9a + 14$$

$$\text{So } a^2 + 9a + 14 = (a + 2)(a + 7)$$

In other words when we factorise  $a^2 + 9a + 14$  we get  $(a + 2)(a + 7)$ .

In this section we will learn how to factorise (find the factors of) quadratic expressions. To do this we can use a method called **The Guide Number Method**.

### Example 1

Factorise  $x^2 + 7x + 12$

#### Step 1: Find the Guide Number (GN)

We find the guide number by multiplying the  $x^2$  coefficient (in this case 1) by the constant (in this case 12)

$$\text{GN} = 1 \times 12 = 12.$$

#### Step 2: Write out all the factors of the Guide Number

Factors of 12 are:

$$1 \times 12, \quad -1 \times -12$$

$$2 \times 6, \quad -2 \times -6$$

$$3 \times 4, \quad -3 \times -4$$

#### Step 3: Pick the factors of 12 which *add* to give you the $x$ coefficient in the original quadratic (in this case 7)

$$3 \times 4$$

#### Step 4: Split the $x$ term in the original quadratic into two terms using the values found in Step 3

$$\begin{aligned} &x^2 + 7x + 12 \\ &= x^2 + 4x + 3x + 12 \end{aligned}$$

#### Step 5: Proceed by factorising by grouping

$$\begin{aligned} &(x^2 + 4x) + (3x + 12) \\ &= x(x + 4) + 3(x + 4) \end{aligned}$$

Factors:  $(x + 4)(x + 3)$ .

**Example 2**Factorise  $a^2 + 9a + 20$ **Step 1: Find the Guide Number (GN)**

We find the guide number by multiplying the  $x^2$  coefficient (in this case 1) by the constant (in this case 20)

$$\text{GN} = 1 \times 20 = 20.$$

**Step 2: Write out all the factors of the Guide Number**

Factors of 20 are:

$$1 \times 20, \quad -1 \times -20$$

$$2 \times 10, \quad -2 \times -10$$

$$4 \times 5, \quad -4 \times -5$$

**Step 3: Pick the factors of 20 which add to give you the  $a$  coefficient in the original quadratic (in this case 9)**

$$4 \times 5$$

**Step 4: Split the  $a$  term in the original quadratic into two terms using the values found in Step 3**

$$\begin{aligned} a^2 + 9a + 20 \\ = a^2 + 4a + 5a + 20 \end{aligned}$$

**Step 5: Proceed by factorising by grouping**

$$\begin{aligned} (a^2 + 4a) + (5a + 20) \\ = a(a + 4) + 5(a + 4) \end{aligned}$$

Factors:  $(a + 4)(a + 5)$ .

### Example 3

Factorise  $3x^2 + 7x + 2$ .

#### Step 1: Find the Guide Number (GN)

We find the guide number by multiplying the  $x^2$  coefficient (in this case 3) by the constant (in this case 2)

$$\text{GN} = 3 \times 2 = 6.$$

#### Step 2: Write out all the factors of the Guide Number

Factors of 6 are:

$$1 \times 6, \quad -1 \times -6$$

$$2 \times 3, \quad -2 \times -3$$

#### Step 3: Pick the factors of 6 which *add* to give you the $x$ coefficient in the original quadratic (in this case 7)

$$1 \times 6$$

#### Step 4: Split the $x$ term in the original quadratic into two terms using the values found in Step 3

$$\begin{aligned} &3x^2 + 7x + 2 \\ &= 3x^2 + x + 6x + 2 \end{aligned}$$

#### Step 5: Proceed by factorising by grouping

$$\begin{aligned} &(3x^2 + x) + (6x + 2) \\ &= x(3x + 1) + 2(3x + 1) \end{aligned}$$

Factors:  $(3x + 1)(x + 2)$ .

**Example 4**Factorise  $x^2 - 7x + 10$ **Step 1: Find the Guide Number (GN)**

We find the guide number by multiplying the  $x^2$  coefficient (in this case 1) by the constant (in this case 10)

$$\text{GN} = 1 \times 10 = 10$$

**Step 2: Write out all the factors of the Guide Number**

Factors of 10 are:

$$1 \times 10, \quad -1 \times -10$$

$$2 \times 5, \quad -2 \times -5$$

**Step 3: Pick the factors of 10 which *add* to give you the  $x$  coefficient in the original quadratic (in this case -7)**

$$-2 \times -5$$

**Step 4: Split the  $x$  term in the original quadratic into two terms using the values found in Step 3**

$$\begin{aligned} & x^2 - 7x + 10 \\ &= x^2 - 2x - 5x + 10 \end{aligned}$$

**Step 5: Proceed by factorising by grouping**

$$(x^2 - 2x) + (-5x + 10)$$

$$= x(x - 2) - 5(x - 2)$$

(We took out -5 here so that the terms in both brackets would be the same).

Factors:  $(x - 5)(x - 2)$ .

### Example 5

Factorise  $3x^2 - 10x - 8$

#### Step 1: Find the Guide Number (GN)

We find the guide number by multiplying the  $x^2$  coefficient (in this case 3) by the constant (in this case -8)

$$\text{GN} = 3 \times -8 = -24$$

#### Step 2: Write out all the factors of the Guide Number

Factors of -24 are:

$$1 \times -24, \quad -1 \times 24$$

$$2 \times -12, \quad -2 \times 12$$

$$3 \times -8, \quad -3 \times 8$$

$$4 \times -6, \quad -4 \times 6$$

#### Step 3: Pick the factors of -24 which *add* to give you the $x$ coefficient in the original quadratic (in this case -10)

$$2 \times -12$$

#### Step 4: Split the $x$ term in the original quadratic into two terms using the values found in Step 3

$$\begin{aligned} &3x^2 - 10x - 8 \\ &= 3x^2 + 2x - 12x - 8 \end{aligned}$$

#### Step 5: Proceed by factorising by grouping

$$\begin{aligned} &(3x^2 + 2x) + (-12x - 8) \\ &= x(3x + 2) - 4(3x + 2) \end{aligned}$$

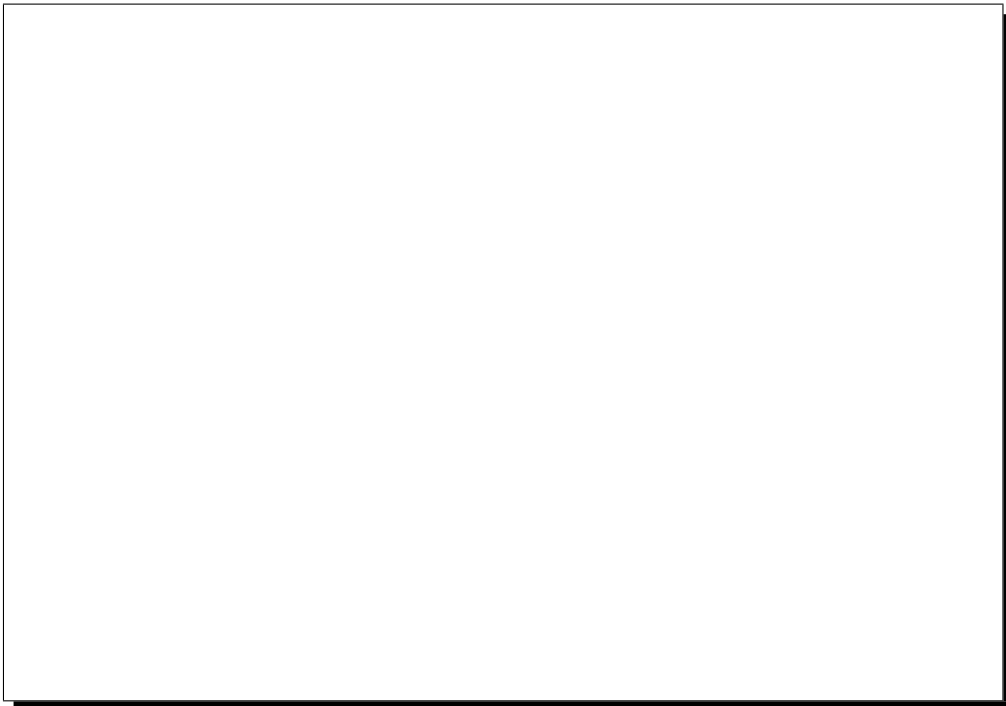
Factors:  $(3x + 2)(x - 4)$ .

**Exercises 3****Factorise the Following**

1.  $x^2 + 10x + 24$



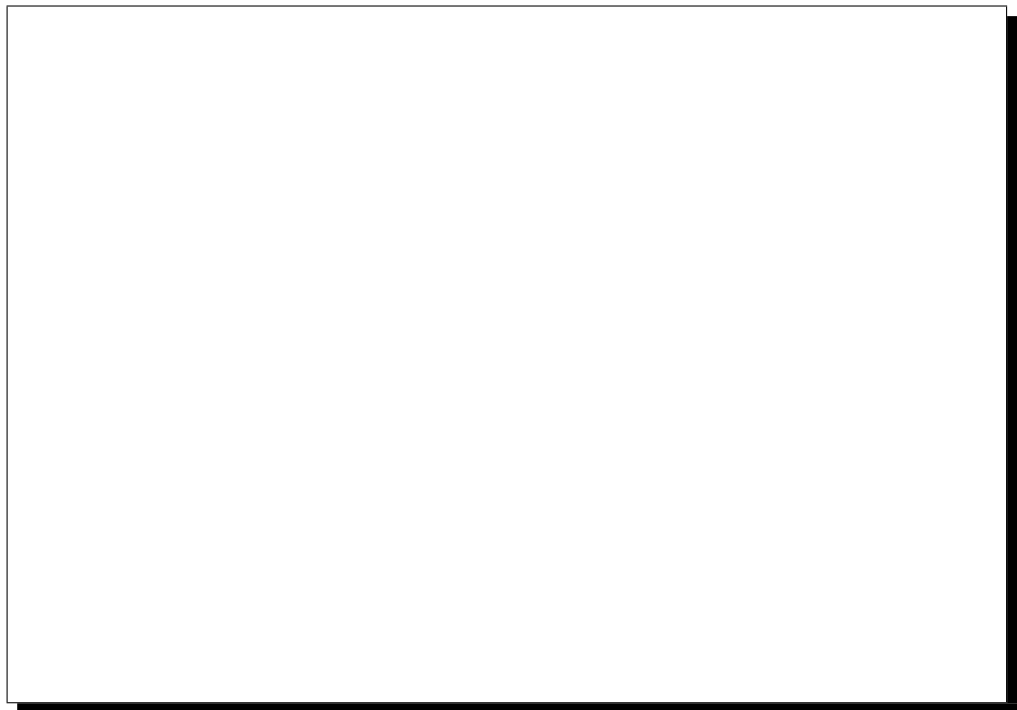
2.  $a^2 + 12a + 27$



3.  $2\beta^2 + 9\beta + 4$



4.  $3\theta^2 + 22\theta - 16$



5.  $2t^2 + 5t - 3$



6.  $b^2 - b - 12$



7.  $3\alpha^2 - 13\alpha - 10$



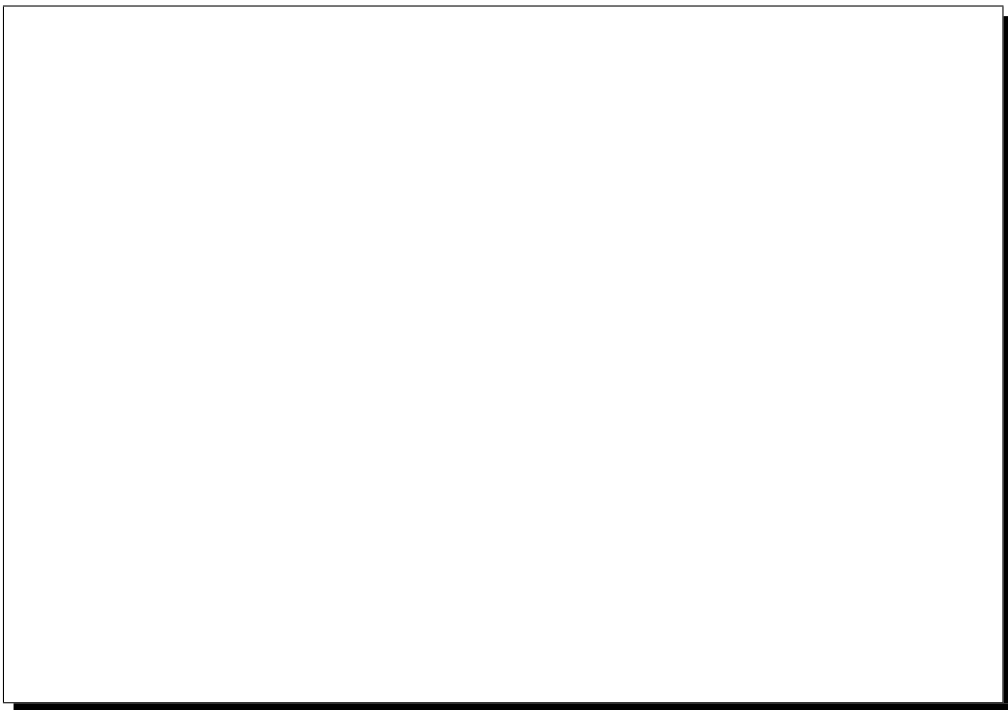
8.  $4z^2 - 13z + 3$



9.  $2q^2 - 9q + 7$



10.  $w^2 - 16w + 64$



## 5.5 Solving Quadratic Equations

### Method 1: Solving by Factorisation

#### Example 1

Solve the following:

$$x^2 + 8x + 15 = 0.$$

Factorise using the guide number method to get

$$(x + 3)(x + 5) = 0$$

The product of two values is zero only if at least one of the values is zero.

$$\text{So we have } (x + 3) = 0 \quad \text{or} \quad (x + 5) = 0$$

$$\text{So we solve to get } x = -3 \quad \text{or} \quad x = -5$$

#### Example 2

Solve the following:

$$3x^2 + x - 2 = 0.$$

Factorise using the guide number method to get

$$(3x - 2)(x + 1) = 0$$

At least one of these values is zero.

$$\text{So we have } (3x - 2) = 0 \quad \text{or} \quad (x + 1) = 0$$

$$\text{So we solve to get } x = \frac{2}{3} \quad \text{or} \quad x = -1$$

## Method 2: Solving by Formula

Not all quadratic equations can be factorised.

For example, solve  $2x^2 + 8x + 3 = 0$

The guide number method will not work here. Why?

The guide number in this instance works out to be  $2 \times 3 = 6$ .

There are no factors of 6 which add up to 8 so the equation does not factorise. For such equations there is a formula we can use. Given a quadratic equation of the form  $ax^2 + bx + c = 0$ ,  $a \neq 0$ , the following formula can be used to find the values of  $x$  that solve the quadratic:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

### Example

Solve  $2x^2 + 8x + 3 = 0$

$a = 2$  ( $x^2$  coefficient),

$b = 8$  ( $x$  coefficient),

$c = 3$  (constant).

Substitute the values for  $a$ ,  $b$  and  $c$  into the formula to solve for  $x$

$$x = \frac{-8 \pm \sqrt{(8)^2 - 4(2)(3)}}{2(2)}$$

$$x = \frac{-8 \pm \sqrt{64 - 24}}{4}$$

$$x = \frac{-8 \pm \sqrt{40}}{4}$$

$$x = \frac{-8 \pm 6.32}{4}$$

So

$$x = \frac{-8 + 6.32}{4} \quad \text{or} \quad x = \frac{-8 - 6.32}{4}$$

$$x = \frac{-1.68}{4} \quad \text{or} \quad x = \frac{-14.32}{4}$$

$$x = -0.42 \quad \text{or} \quad x = -3.58$$

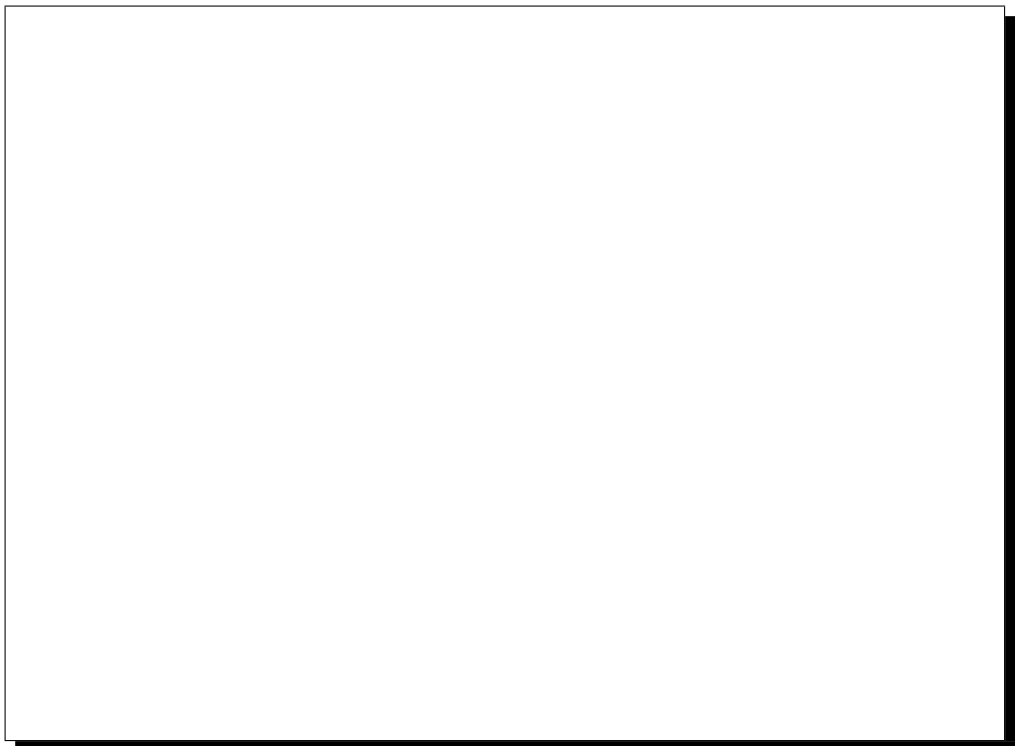
## Exercises 4

Solve the Following Quadratic Equations

1.  $x^2 - x - 6 = 0$  (By Guide Number method)



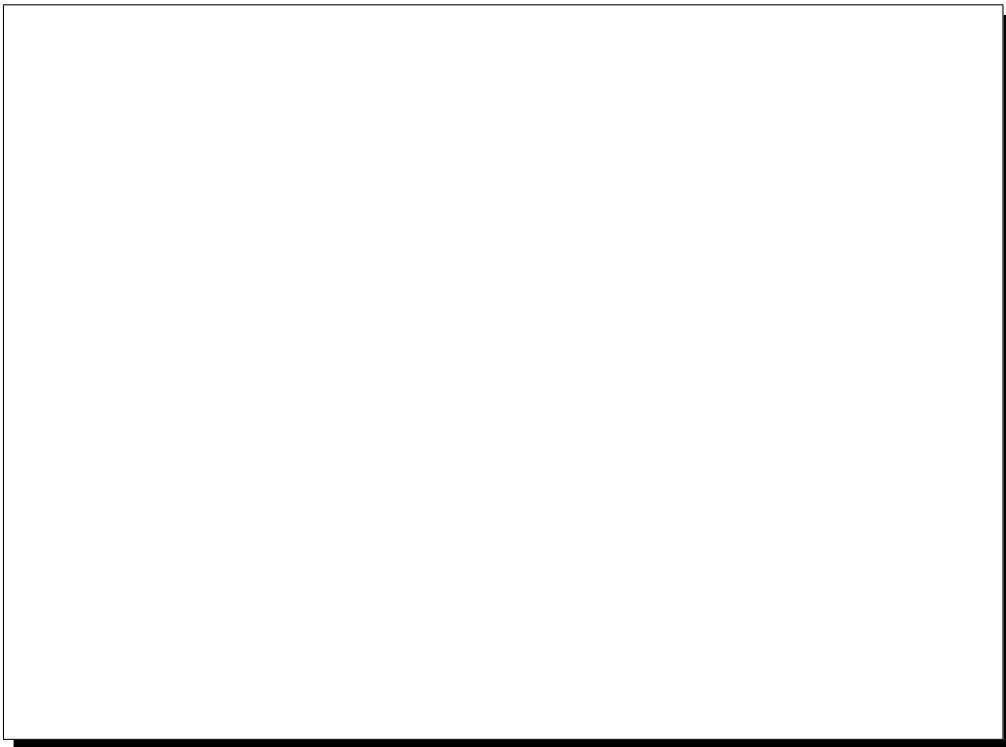
2.  $2k^2 - 3k - 2 = 0$  (By Guide Number method)



3.  $\delta^2 - 3\delta - 10 = 0$  (By formula)



4.  $3\theta^2 - 4\theta + 1 = 0$  (By formula)



## 5.6 Answers

### Exercises 1:

- |   |                                 |
|---|---------------------------------|
| 1). $6(s + 5t)$                                 | 2). $21(u - 2v)$                |
| 3). $a(-b + c)$                                 | 4). $5\beta(5\alpha - 7\delta)$ |
| 5). $3\theta(7\theta^2 - 8)$                    | 6). $11q^2(3q^2 - 2)$           |
| 7). $-25\alpha\beta^2\gamma(1 - 2\alpha\gamma)$ | 8). $-4x(x^4 + 4x^3z + 2y^3)$   |
| 9). $abc(1 + a^2b^3c - a^5b^9c)$                | 10). $n(l^2mn - m^2 + ln)$      |

### Exercises 2:

- |                                      |                                    |
|--------------------------------------|------------------------------------|
| 1). $(q + 4)(x + y)$                 | 2). $(2b + 17)(c - d)$             |
| 3). $(\rho + \lambda)(\pi + \kappa)$ | 4). $(5s + 2q)(a - 2b)$            |
| 5). $(t + 16)(p - 2)$                | 6). $(\alpha - \phi)(2l + \delta)$ |
| 7). $(u - c)(v + 2d)$                | 8). $(2 + \kappa)(\mu - 3\lambda)$ |
| 9). $(h - 4j)(i - 2g)$               | 10). $(16 - r)(s - 2)$             |

### Exercises 3:

- |                                 |                                 |
|---------------------------------|---------------------------------|
| 1). $(x + 6)(x + 4)$            | 2). $(a + 9)(a + 3)$            |
| 3). $(\beta + 4)(2\beta + 1)$   | 4). $(3\theta - 2)(\theta + 8)$ |
| 5). $(2t - 1)(t + 3)$           | 6). $(b + 3)(b - 4)$            |
| 7). $(3\alpha + 2)(\alpha - 5)$ | 8). $(z - 3)(4z - 1)$           |
| 9). $(q - 1)(2q - 7)$           | 10). $(w - 8)^2$                |

### Exercises 4:

- $x = 3$  or  $x = -2$
- $k = -\frac{1}{2}$  or  $k = 2$
- $\delta = 5$  or  $\delta = -2$
- $\theta = 1$  or  $\theta = \frac{1}{3}$





