

Bounded Sequence

Aim

To define what a bounded sequence is.

Learning Outcomes

At the end of this section you will:

- Understand what a bounded sequence is,
- Know how to tell if a sequence is bounded.

Look at the following sequence,

$$a_n = \begin{cases} 1 + \frac{1}{2^n}, & n \text{ odd;} \\ 2^n, & n \text{ even.} \end{cases}$$

The first few terms of the sequence are

$$\begin{aligned} a_1 &= 1 + \frac{1}{2} = \frac{3}{2} && (n = 1, \text{ odd}) \\ a_2 &= 2^2 = 4 && (n = 2, \text{ even}) \\ a_3 &= 1 + \frac{1}{2^3} = 1 + \frac{1}{8} = \frac{9}{8} && (n = 3, \text{ odd}) \\ a_4 &= 2^4 = 16 && (n = 4, \text{ even}) \dots \text{ etc.} \end{aligned}$$

Sequence is $\frac{3}{2}$, 4, $\frac{9}{8}$, 16, ...

It is not so obvious what the next term is. This is why we need the function above to be defined precisely.

The last example is an example of a sequence which is called *unbounded*, i.e. the n^{th} term in the sequence grows without bound.

Definition: A sequence a_1, a_2, a_3, \dots is *bounded* if there exist a number $M > 0$ such that

$$|a_n| < M$$

for every natural number n . This means that regardless of what term we are looking at, the absolute value of that term must be less than M .

In the previous example it is impossible to find a number M that is larger than 2^n for all n and so this sequence is unbounded. Regardless of the value of M that you choose it is still possible to find a value of n such that $2^n > M$.

Look at the following example,

$$\{2^{-n}\}_{n=1}^{\infty}$$

is an example of a bounded sequence. It is bounded above by 1 - technically any value greater than $\frac{1}{2}$ will bound this sequence. List the first few terms to convince yourself that it is in fact bounded above by 1.

Related Reading

Gersting, J.L. 2007. *Mathematical Structures for Computer Science*. 6th Edition. Freeman & Company.

Morris, O.D. 1992. *Text & Tests 4*. The Celtic Press.