Complete Graphs

Aim

To introduce the ideas of complete and bipartite complete graphs.

Learning Outcomes

At the end of this section you will:

- Understand what complete and bipartite complete graphs are,
- Know how to represent complete and bipartite complete graphs graphically,
- Know how to denote complete and bipartite complete graphs using notation.

Recall that a *complete graph* is a graph in which every pair of vertices is adjacent e.g. a triangle. The following figure represents the simple, complete graphs with 1,2,3 and 4 vertices. The simple, complete graph with n vertices is denoted K_n.

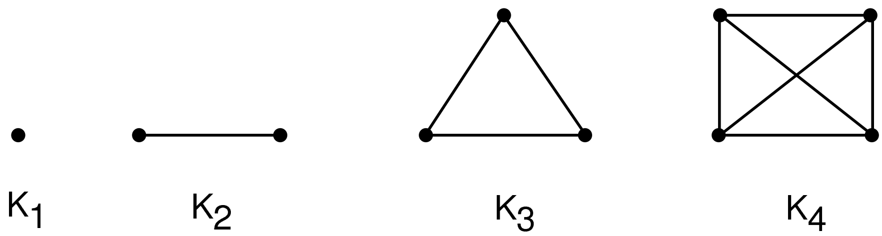

Figure 1: K_1, K_2, K_3 and K_4

Consider the graph in Figure 2. It is not a complete graph because it is not true that every vertex is adjacent to every other vertex. However, the vertices can be divided into two disjoint sets, $\{1, 2\}$ and $\{3, 4, 5\}$, such that any two vertices chosen from the same set are not adjacent but any two vertices chosen one from each set are adjacent. Such a graph is a *bipartite complete graph.*

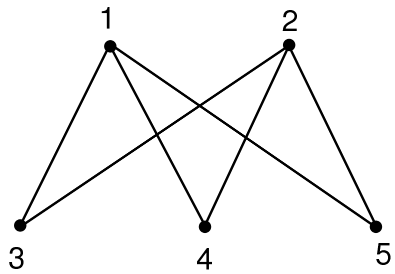

Figure 2: An example of a bipartite graph

We can define a bipartite complete graph as follows:

Bipartite Complete Graph: A graph is a *bipartite complete graph* if its vertices can be partitioned into two disjoint nonempty sets V_1 and V_2 such that two vertices x and y are adjacent if and only if $x \in V_1$ and $y \in V_2$. If $|V_1| = m$ and $|V_2| = n$, such a graph is denoted $K_{m,n}$.

Therefore, the graph in Figure 2 is $K_{2,3}$.

Related Reading

Gersting, J.L. 2007. *Mathematical Structures For Computer Science.* W.H. Freeman and Company.