

## Computer Representation

### Aim

To demonstrate how computers represent graphs.

### Learning Outcomes

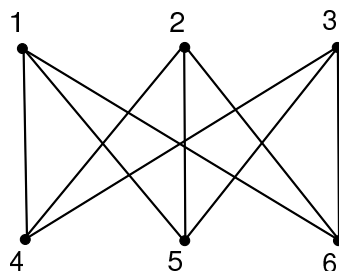
At the end of this section you will:

- Know a possible way that computers represent graphs,
- Know what is meant by the adjacency matrix of a graph,
- Know how to construct the adjacency matrix of a graph.

We can see that the major advantage of a graph is its visual representation of information. What if we wanted to store a graph in digital form? We could store it as a digital image of the graph, but this takes a lot of space. What we need to store are the essential data that are part of the graph — what the vertices are and which vertices have connecting edges. From this information a visual representation could be reconstructed if desired. The computer representation method that we shall look at is called an adjacency matrix.

Suppose a graph has  $n$  vertices, numbered  $n_1, n_2, n_3, \dots, n_n$ . Having ordered the nodes, we can form an  $n \times n$  matrix where entry  $ij$  is the number of edges between vertices  $n_i$  and  $n_j$ . This matrix is called the *adjacency matrix* of the graph with respect to this ordering.

Write down the adjacency matrix of the following graph.



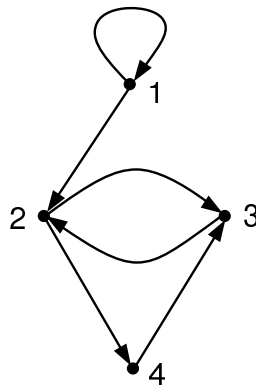
Adjacency Matrix ( $A$ ) is

$$A = \begin{bmatrix} 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 \end{bmatrix}$$

The adjacency matrix is symmetric, which will be true for any undirected graph — if there are  $p$  edges between  $n_i$  and  $n_j$  then there are certainly  $p$  edges between  $n_j$  and  $n_i$ . The symmetry of the matrix means that only elements on or below the main diagonal need to be stored. Therefore, all the information contained in the graph is contained in the “lower triangular” section of the matrix, and the graph could be reconstructed from this array.

The adjacency matrix for a directed graph is not symmetric.

Problem: Construct the adjacency matrix for the following directed graph.



## Related Reading

Gersting, J.L. 2007. *Mathematical Structures For Computer Science*. W.H. Freeman and Company.