The Limit of a Sequence

Aim

To show how to calculate the limit of a sequence.

Learning Outcomes

At the end of this section you will:

- Know how to calculate the limit of a sequence.

We have already seen that all convergent sequences have a limit, that is, the sequence converges to a specific value.

In order for a given sequence $\{a_n\}_{n=1}^{\infty}$ to converge to a limit,

$$a_n \to L \text{ as } n \to \infty \qquad (L = \text{some number}),$$

what we really mean is that the distance between a_n and L becomes almost nothing as n is increased to infinity,

$$|a_n - L| \to 0 \quad \text{as} \quad n \to \infty.$$

Not all sequences have a limit. To find the limit of a sequence, if it exists, we must calculate

$$\lim_{n \to \infty} a_n$$

We have already seen the procedure for calculating the limit of a sequence - divide each term in the sequence by the highest power of "x" in the sequence and then take the limit of the resulting sequence.

Example

$$\begin{aligned}
\lim_{x \to \infty} \frac{x^3}{x^2 - 4} &= \lim_{x \to \infty} \frac{\frac{x^3}{x^3}}{\frac{x^2}{x^3} - \frac{4}{x^3}} \\
&= \lim_{x \to \infty} \frac{1}{\frac{1}{x} - \frac{4}{x^3}} = \frac{1}{\frac{1}{\infty} - \frac{4}{\infty}} \\
&= \frac{1}{0 + 0} = \infty
\end{aligned}$$

Example

$$\begin{aligned}\lim_{x\to\infty}\frac{x^2+3}{x^3-4} &= \lim_{x\to\infty}\frac{\frac{x^2}{x^3}+\frac{3}{x^3}}{\frac{x^3}{x^3}-\frac{4}{x^3}}\\ &= \lim_{x\to\infty}\frac{\frac{1}{x}+\frac{3}{x^3}}{1-\frac{4}{x^3}} = \frac{\frac{1}{\infty}+\frac{3}{\infty}}{1-\frac{4}{\infty}}\\ &= \frac{0+0}{1-0} = \frac{0}{1} = 0\end{aligned}$$

Related Reading

Gersting, J.L. 2007. *Mathematical Structures for Computer Science.* 6^{th} Edition. Freeman & Company.