

Sigma Notation

Aim

To introduce the sigma notation.

Learning Outcomes

At the end of this section you will:

- Understand what the sigma notation is,
- Understand how the sigma notation works.

The Greek letter \sum (pronounced *sigma*), which means *the sum of*, is generally used to express a series in a concise way.

$$2 + 4 + 8 + 16 + 32 = 2 + 2^2 + 2^3 + 2^4 + 2^5 = \sum_{r=1}^{r=5} 2^r$$

Notice 2^r is the r^{th} term in the sequence. Since we are summing the terms from 1 to 5 inclusive, the least value of r is placed below the sigma sign and the greatest value of r is placed above.

A finite series always ends with the last term even if several middle terms are omitted.

$$\sum_{r=1}^{r=20} \frac{1}{2r+2} = \frac{1}{4} + \frac{1}{6} + \frac{1}{8} + \dots + \frac{1}{42}$$

An infinite series may also be written in sigma notation, where ∞ is used to indicate that there is no upper limit for r .

$$2 + 4 + 6 + 8 + \dots = \sum_{r=1}^{\infty} 2r$$

Example

Write the following series using \sum notation:

1. $1 + 4 + 9 + 16 + \dots + 81$
2. $1 - x + x^2 - x^3 + x^4 - \dots$

$$1 + 4 + 9 + 16 + \dots + 81 = 1 + 2^2 + 3^2 + \dots + 9^2 = \sum_{r=1}^{r=9} r^2$$

In a series where the sign alternates from positive to negative, $(-1)^r$ may be used for the sign. If for example the counter r started at $r = 1$ then $(-1)^r$ would result in even terms being positive and odd terms being negative whereas $(-1)^{r+1}$ would result in even terms being negative and odd terms being positive.

$$1 - x + x^2 - x^3 + x^4 - \dots = \sum_{r=0}^{\infty} (-1)^r x^r$$

Related Reading

Gersting, J.L. 2007. *Mathematical Structures for Computer Science*. 6th Edition. Freeman & Company.

Morris, O.D. 1992. *Text & Tests 4*. The Celtic Press.