S_N Notation

Aim

To introduce the S_N notation.

Learning Outcomes

At the end of this section you will:

- Understand what the S_N notation is,
- Know how to calculate the sum of N terms of a series.

S_N is generally used to denote the sum to N terms of a series. In the series $1+3+5+7+9+...,\quad S_4=16$. That is, the sum of the first 4 terms is 16.

In general $S_N = a_1 + a_2 + a_3 + a_4 + ... + a_N$

$$\Rightarrow S_N = \sum_{r=1}^{N} a_r$$

<u>Example</u>

Consider the sequence $a_r = \dfrac{x^{r-1}}{(r-1)!}$ for any given x.

$$\{a_r\} = 1, x, \frac{x^2}{2!}, \frac{x^3}{3!}, \frac{x^4}{4!}, ...$$

$$\begin{aligned} S_N(x) &= \sum_{r=1}^{N} a_r \\ &= 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + ... \end{aligned}$$

Remember that x is a fixed real number. S_1, S_2, S_3, ... define a sequence of numbers that are defined in terms of x. i.e. $1,\ 1+x,\ 1+x+\dfrac{x^2}{2!},\ 1+x+\dfrac{x^2}{2!}+\dfrac{x^3}{3!}, ...$

This is in fact a converging sequence whose limit is e^x. Therefore it is possible to define a limit in terms of a series,

$$S_\infty(x) = \sum_{r=1}^{N} a_r = e^x \quad (\text{ in this example})$$

Definition: If a sequence $\{a_r\}$ converges to a limit ℓ we can write

$$\ell = \sum_{r=1}^{\infty} a_r$$

To test and see if a series converges we use the ratio test - see next section.

Related Reading

Gersting, J.L. 2007. *Mathematical Structures for Computer Science.* 6^{th} Edition. Freeman & Company.

Morris, O.D. 1992. *Text & Tests 4.* The Celtic Press.