Addition and Subtraction of Matrices

Aim

To demonstrate how to add and subtract matrices.

Learning Outcomes

At the end of this section you will be able to:

- Add matrices,
- Subtract matrices,
- Find the transpose of a matrix.

A matrix is a rectangular array of numbers arranged in rows and columns, the array being enclosed in round brackets. Each number in the array is called an **element** or **entry**. Matrices are generally denoted by capital letters. Some examples of matrices are shown below:

$$A = \begin{pmatrix} 1 & 4 \\ -2 & 4 \end{pmatrix}; \quad B = \begin{pmatrix} 3 & 6 \end{pmatrix}; \quad C = \begin{pmatrix} 2 & 6 & -2 \\ 7 & 2 & 3 \end{pmatrix}$$

The Order of a Matrix

The order of a matrix is given by stating the number of **rows** followed by the number of **columns**. When the number of rows equals the number of columns the matrix is said to be a **square matrix**. The following is the order of the three matrices presented above;

$$A = (2 \times 2) \text{ matrix}, \quad B = (1 \times 2) \text{ matrix}, \quad C = (2 \times 3) \text{ matirx}.$$

Equal Matrices

Two matrices are equal if

- they are of the same order and
- their corresponding entries are equal.

Position within a Matrix

We can determine the position of an element within a matrix by listing its location in terms of the row and column that it is located in. In the matrix below, the element P_{ij} represents the entry in position i, j of the matrix, where i is the row number and j is the column number.

$$A = \begin{pmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{pmatrix}$$

Addition and Subtraction

If two matrices are to be added or subtracted, they must be of the same order. If A and B are two matrices of the same order, then $A+B$ and $A-B$ are found by adding and subtracting the corresponding elements.

Example

If $A = \begin{pmatrix} 2 & 5 \\ 3 & -4 \end{pmatrix}$ and $B = \begin{pmatrix} -1 & 7 \\ 0 & 9 \end{pmatrix}$, find $A+B$ and $A-B$.

$$\begin{aligned} A+B &= \begin{pmatrix} 2+(-1) & 5+7 \\ 3+0 & -4+9 \end{pmatrix} = \begin{pmatrix} 1 & 12 \\ 3 & 5 \end{pmatrix}, \\ A-B &= \begin{pmatrix} 2-(-1) & 5-7 \\ 3-0 & -4-9 \end{pmatrix} = \begin{pmatrix} 3 & -2 \\ 3 & -13 \end{pmatrix}. \end{aligned}$$

Note: For matrices A and B of the same order

- $A+B = B+A$, and
- $(A+B)+C = A+(B+C)$.

A Zero Matrix

A zero matrix, denoted by O, is one in which all the elements are zero.

The Transpose of a Matrix

The transpose of a matrix A, written A^T, is obtained by interchanging the rows and the columns of A.

If $A = \begin{pmatrix} 2 & 3 \\ 5 & 1 \end{pmatrix}$ then $A^T = \begin{pmatrix} 2 & 5 \\ 3 & 1 \end{pmatrix}$.

If the order of the matrix B is $n \times m$, then the order of B^T is $m \times n$.

Related Reading

Morris, O.D., P. Cooke. 1993. *Text & Tests 5.* The Celtic Press.