Subtracting and Adding Vectors

Aim

To show how to subtract and add vectors.

Learning Outcomes

At the end of this section you will be able to:

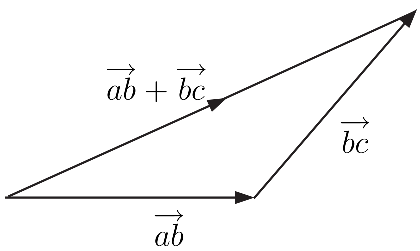
- Subtract/add one vector from/to another,
- Subtract and add vectors written in component form.

The Laws of Subtracting and Adding Vectors

Vectors may be added using the **triangle law** or the **parallelogram law** as demonstrated below.

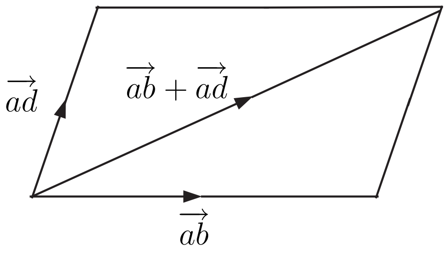
Triangle Law

In the triangle given below, the vector $\overrightarrow{ab}$ is followed by the vector $\overrightarrow{bc}$. The third side of the triangle, ($\overrightarrow{ac}$) represents the sum of the vectors $\overrightarrow{ab}$ and $\overrightarrow{bc}$. This may be written as $\overrightarrow{ab} + \overrightarrow{bc} = \overrightarrow{ac}$. Notice that the vector $\overrightarrow{ac}$ goes from the starting point of the first vector to the end point of the second vector.



Parallelogram Law

If the starting point of two vectors $\overrightarrow{ab}$ and $\overrightarrow{ad}$ coincide as shown on the following page, then the diagonal of the parallelogram formed by these vectors will represent the sum of these two vectors: $\overrightarrow{ab} + \overrightarrow{ad}$.



Vector subtraction is defined in terms of addition and scalar multiplication by

$$\overrightarrow{v} - \overrightarrow{w} = \overrightarrow{v} + (-\overrightarrow{w}).$$

The method here is to first of all form the vector $-\overrightarrow{w}$ (the negative of the vector $\overrightarrow{w}$) and then add this new vector (using the parallelogram or triangle law) to the vector $\overrightarrow{v}$.

Vectors in Coordinate Systems

If a vector $\overrightarrow{v}$ is positioned so that its initial point is at the origin of a rectangular coordinate system, then its end point will have the form (v_1, v_2) (2-D space) or (v_1, v_2, v_3) (3-D space). We call these coordinates the **components** of $\overrightarrow{v}$, and we can write $\overrightarrow{v}$ in *component form* as

$$\overrightarrow{v} = \langle v_1, v_2 \rangle \qquad \text{or} \qquad \overrightarrow{v} = \langle v_1, v_2, v_3 \rangle.$$

The components provide a simple way or identifying equivalent vectors. If two vectors, $\overrightarrow{v} = \langle v_1, v_2 \rangle$ and $\overrightarrow{w} = \langle w_1, w_2 \rangle$, are equal then that would mean that they have the same length and the same direction. This would mean that their end points coincide when their initial points are placed at the origin. It follows that $v_1 = w_1$ and $v_2 = w_2$, so we have shown that equivalent vectors have the same components.

Theorem: Two vectors are equivalent if and only if their corresponding components are equal.

Arithmetic Operations on Vectors written in Component Form

The following results shall be shown for 2-D space but similar results are possible in 3-D space too.

Theorem: If $\overrightarrow{v} = \langle v_1, v_2 \rangle$ and $\overrightarrow{w} = \langle w_1, w_2 \rangle$ are vectors in 2-D space and k is any scalar, then

$$\overrightarrow{v} + \overrightarrow{w} = \langle v_1 + w_1, v_2 + w_2 \rangle,$$

$$\overrightarrow{v} - \overrightarrow{w} = \langle v_1 - w_1, v_2 - w_2 \rangle,$$

$$k\overrightarrow{v} = \langle kv_1, kv_2 \rangle.$$

Example

If $\overrightarrow{v} = \langle -2, 0, 1 \rangle$ and $\overrightarrow{w} = \langle 1, -3, 5 \rangle$ then,

$$\begin{aligned} \overrightarrow{v} + \overrightarrow{w} &= \langle -2+1,\ 0+(-3),\ 1+5 \rangle = \langle -1, -3, 6 \rangle, \\ \overrightarrow{v} - \overrightarrow{w} &= \langle -2-1,\ 0-(-3),\ 1-5 \rangle = \langle -3, 3, -4 \rangle, \\ 3\overrightarrow{w} &= \langle 3(1),\ 3(-3),\ 3(5) \rangle = \langle 3, -9, 15 \rangle. \end{aligned}$$

Related Reading

Adams, R.A. 2003. *Calculus: A Complete Course.* 5^{th} Edition. Pearson Education Limited.

Anton, H., I. Bivens, S. Davis. 2005. *Calculus.* 8^{th} Edition. John Wiley & Sons.

Morris, O.D., P. Cooke. 1993. *Text & Tests 5.* The Celtic Press.