

Applications of Vectors

Aim

To demonstrate the application of vectors in the real world.

Learning Outcomes

At the end of this section you will be able to:

- Tell the difference between a resultant and a concurrent force,
- Apply vectors to a real world problem.

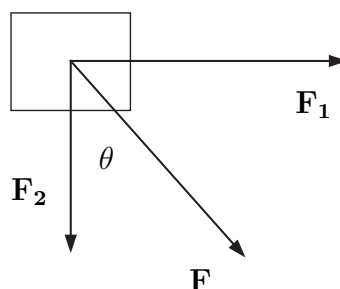
The Resultant of two Concurrent Forces

The effect that a force has on an object depends on the magnitude and direction of the force and the point at which it is applied. Thus, forces are regarded to be vector quantities. It is a fact of physics that if two forces \mathbf{F}_1 and \mathbf{F}_2 are applied at the same point on an object, then the two forces have the same effect on the object as a single force $\mathbf{F}_1 + \mathbf{F}_2$ applied at the same point. Physicists and engineers call $\mathbf{F}_1 + \mathbf{F}_2$ the **resultant** of \mathbf{F}_1 and \mathbf{F}_2 , and they say that the forces \mathbf{F}_1 and \mathbf{F}_2 are **concurrent** to indicate that they are applied at the same point.

The cosine rule, the sine rule and the basic rules of trigonometry are used when solving problems of this type.

Example

An object is experiencing two perpendicular forces $\mathbf{F}_1 = 50N$ and $\mathbf{F}_2 = 30N$ as shown in the diagram.



Calculate the magnitude of the resultant force \mathbf{F} and determine the angle θ that \mathbf{F} makes with the vertical axis.

$$\begin{aligned} |\mathbf{F}|^2 &= |\mathbf{F}_1|^2 + |\mathbf{F}_2|^2, && \text{Pythagoras' Theorem} \\ \Rightarrow |\mathbf{F}|^2 &= (50)^2 + (30)^2, \\ \Rightarrow |\mathbf{F}|^2 &= 3400, \\ \Rightarrow \mathbf{F} &= \sqrt{3400} \approx 58.3N. \end{aligned}$$

To calculate the angle θ use the fact that $\tan \theta = \frac{50}{30}$. Therefore $\theta = \tan^{-1} \frac{50}{30} = 59^\circ 02'$.

Related Reading

Adams, R.A. 2003. *Calculus: A Complete Course*. 5th Edition. Pearson Education Limited.

Anton, H., I. Bivens, S. Davis. 2005. *Calculus*. 8th Edition. John Wiley & Sons.