

## Preliminary Knowledge

### Aim

To present some necessary definitions for dealing with systems of linear equations.

### Learning Outcomes

At the end of this section you will be able to:

- Understand the terminology of this area,
- Write a system of linear equations as an augmented matrix.

A finite set of linear equations in the variables  $x_1, x_2, \dots, x_n$  is called a **system of equations** or a **linear system**. A sequence of numbers  $s_1, s_2, \dots, s_n$  is called a **solution** of the system if  $x_1 = s_1, x_2 = s_2, \dots, x_n = s_n$  is a solution of every equation in the system. The set of **all** solutions is called the **solution set**.

An arbitrary system of  $m$  linear equations in  $n$  unknowns can be written as

$$\begin{aligned} a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n &= b_1 \\ a_{21}x_1 + a_{22}x_2 + \dots + a_{2n}x_n &= b_2 \\ \cdot & \quad \cdot & \quad \cdot & \quad \cdot \\ \cdot & \quad \cdot & \quad \cdot & \quad \cdot \\ a_{m1}x_1 + a_{m2}x_2 + \dots + a_{mn}x_n &= b_m \end{aligned}$$

If we mentally keep track of the location of the +’s, the  $x$ ’s and the =’s, a system of  $m$  linear equations in  $n$  unknowns can be abbreviated by writing only the rectangular array of numbers:

$$\left( \begin{array}{cccc|c} a_{11} & a_{12} & \dots & a_{1n} & b_1 \\ a_{21} & a_{22} & \dots & a_{2n} & b_2 \\ \cdot & \cdot & & \cdot & \cdot \\ \cdot & \cdot & & \cdot & \cdot \\ a_{m1} & a_{m2} & \dots & a_{mn} & b_m \end{array} \right)$$

This is called the **augmented matrix** of the system. Since the rows of the augmented matrix correspond to the equations in the associated system, the basic operations

for solving systems of equations correspond to the following operations on the rows of the augmented matrix;

- Multiply a row through by a nonzero constant,
- Interchange two rows,
- Add a multiple of one row to another row.

These are called **elementary row operations**.

### Related Reading

Anton, H. 1994. *Elementary Linear Algebra*. 7<sup>th</sup> Edition. John Wiley & Sons Inc.