

## Determinants

### Aim

To demonstrate how to calculate the determinant of a matrix.

### Learning Outcomes

At the end of this section you will be able to:

- Calculate the determinant of a  $(2 \times 2)$  matrix,
- Calculate the determinant of a  $(3 \times 3)$  matrix.

### The Determinant of a $(2 \times 2)$ Matrix

Determinants are functions that assign numerical values to square arrays of numbers (square matrices). For example, if the  $2 \times 2$  matrix  $A$  is made up of the real numbers  $a, b, c$  and  $d$  then we define the **determinant** by

$$\det(A) \text{ or } |A| = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc.$$

### The Determinant of a $(3 \times 3)$ Matrix

If  $A$  is a  $3 \times 3$  matrix, then its **determinant** is defined in terms of  $2 \times 2$  determinants by

$$\det(A) \text{ or } |A| = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = a_1 \begin{vmatrix} b_2 & b_3 \\ c_2 & c_3 \end{vmatrix} - a_2 \begin{vmatrix} b_1 & b_3 \\ c_1 & c_3 \end{vmatrix} + a_3 \begin{vmatrix} b_1 & b_2 \\ c_1 & c_2 \end{vmatrix}.$$

The right hand side of this formula is easily remembered by noting that  $a_1, a_2$  and  $a_3$  are the entries in the first “row” of the left hand side, and the  $2 \times 2$  determinants on the right hand side arise by deleting the first row and an appropriate column (depending on which  $a$  case is being looked at) from the left hand side.

### Example

Find the determinant of  $A$  when  $A = \begin{pmatrix} 0 & 1 & 5 \\ 3 & -6 & 9 \\ 2 & 6 & 1 \end{pmatrix}$ .

$$\begin{aligned}\det(A) &= 0 \begin{vmatrix} -6 & 9 \\ 6 & 1 \end{vmatrix} - 1 \begin{vmatrix} 3 & 9 \\ 2 & 1 \end{vmatrix} + 5 \begin{vmatrix} 3 & -6 \\ 2 & 6 \end{vmatrix}, \\ &= 0[-6(1) - 9(6)] - 1[3(1) - 2(9)] + 5[3(6) - (-6)2], \\ &= 0[-60] - 1[-15] + 5[30], \\ &= 0 + 15 + 150, \\ &= 165.\end{aligned}$$

## Related Reading

Anton, H. 1994. *Elementary Linear Algebra*. 7<sup>th</sup> Edition. John Wiley & Sons Inc.