Inverse Functions

Aim

To demonstrate how to calculate the inverse function, if it exists.

Learning Outcomes

At the end of this section you will be able to:

- Understand what is meant by the inverse of a function,
- Know how to graph the inverse of a function,
- Calculate the inverse function, if it exists.

A function is a process f that converts an input x to an output number y. If the input and the output are reversed so that y now becomes the input number and x becomes the output the action of the process is also reversed. When the process is reversed, it becomes a totally different process and is labelled f^{-1}.

Note that this is "f inverse" and not "f to the power of -1". Also note that f^{-1} is not $\frac{1}{f}$. There can be some confusion when working with the inverse function but remember that this new function is in no way related to the reciprocal of f.

In general, if $f(x) = y$ then $f^{-1}(y) = x$ provided that f^{-1} exists.

Example

Find the inverse of $y = 4x - 2$.

The process is to first isolate x, i.e. rewrite the given function in terms of x instead of y.

$$\begin{aligned} y &= 4x - 2, \\ y + 2 &= 4x, \\ \frac{y+2}{4} &= x. \end{aligned}$$

The final step in finding the inverse once you have isolated x from the original function is to switch y and x around so that you get a new function in terms of x. Therefore the final answer will be

$$f^{-1}(x) = y = \frac{x+2}{4}.$$

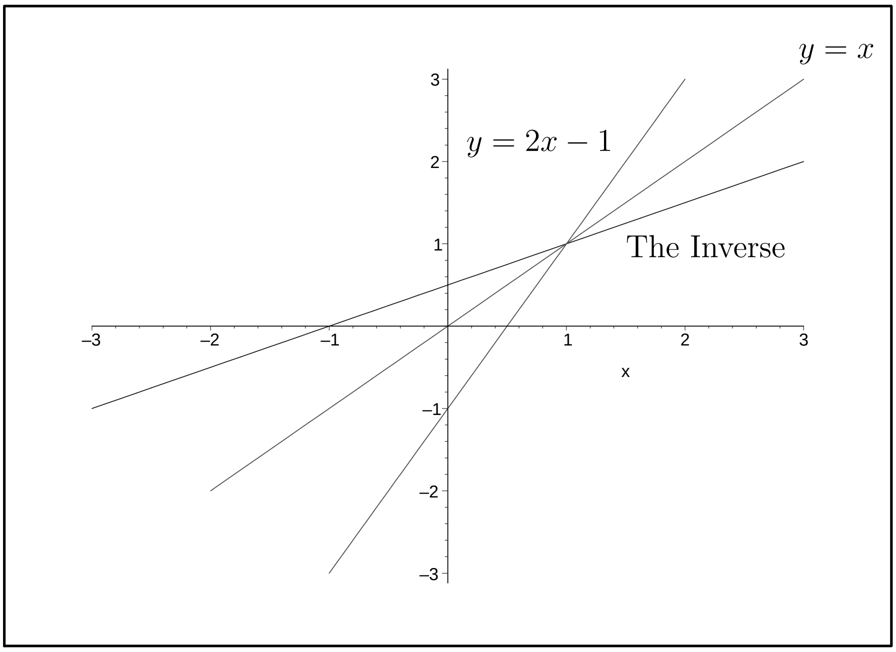
Graphing the Inverse of a Function

The inverse of a function has all the same points, except that the x and y coordinates have been reversed. From this it can be concluded that the inverse is "a reflection in the line $y = x$". This is true because reflecting any point through the line $y = x$ only reverses the x and y coordinates of the point. Therefore, graphically, the inverse of any function can be found by simply reflecting all the points through the line $y = x$.

Example 2

Graph the function $y = 2x - 1$ and then from the graph find its inverse over the domain $[-3, 3]$.

Using the same approach from the section on "Graphing Functions" it is possible to graph the linear function $y = 2x - 1$ and then by inserting the line $y = x$ on the graph and reflection the points through $y = x$ it is possible to graphically come up with the inverse function of $y = 2x - 1$ without specifically evaluating it.



Does the Inverse Function Exist?

Although it is possible to compute the inverse of almost every function, not all of these inverses are functions. Take for example the function $y = x^2$. If 3 is the input then the output is 9. Also if -3 is the input the output will also be 9 since $(-3)^2 = 9$. In order to reverse this process an inverse would have to take an input of 9 and produce outputs of 3 and -3. However, this contradicts the definition of a function, which states that a function must have only *one* output for a given input. Therefore it can be said that $y = x^2$ does not have an inverse function.

Related Reading

Booth, D.J. 1998. *Foundation Mathematics.* 3^{rd} Edition. Pearson Education Limited.

Croft, A., R. Davison. 2003. *Foundation Mathematics.* 3^{rd} Edition. Pearson Education Limited.