Multiplication of Matrices

Aim

To demonstrate how to multiply matrices.

Learning Outcomes

At the end of this section you will be able to:

- Multiply a matrix by a scalar,
- Multiply two matrices.

Multiplication of a Matrix by a Scalar

If k is a real number (scalar) and A is a matrix, then kA is the matrix obtained by multiplying each entry of A by k.

If $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ then $kA = \begin{pmatrix} ka & kb \\ kc & kd \end{pmatrix}$.

Multiplication of Matrices

Two matrices A and B may be multiplied only if the number of columns of A equals the number of rows of B. In general, if A is a $m \times n$ matrix and B is a $n \times p$ matrix, then AB can be multiplied and the order of the resulting product matrix AB will be $m \times p$.

To multiply two matrices A and B, to get AB, we multiply the **rows** of A by the **columns** of B. Note: The order of the matrices is important, i.e. whether A comes before B or B before A.

If we multiply row 1 of matrix A by column 1 of matrix B, then the resulting value will be stored in position 1,1 of the matrix AB. Similarly if we multiplied row 2 of matrix A by column 1 of matrix B then the resulting value would be stored in position 2,1 of matrix AB. Following this pattern it is possible to conclude that in general if we multiply row i by column j the resulting value will be stored in position i, j of the product matrix.

Example

If $A = \begin{pmatrix} 5 & 3 \\ 2 & -1 \end{pmatrix}$ and $B = \begin{pmatrix} 2 & 1 \\ 6 & 4 \end{pmatrix}$, find AB.

$$\begin{aligned} AB = \begin{pmatrix} 5 & 3 \\ 2 & -1 \end{pmatrix}\begin{pmatrix} 2 & 1 \\ 6 & 4 \end{pmatrix} &= \begin{pmatrix} 5(2)+3(6) & 5(1)+3(4) \\ 2(2)+(-1)(6) & 2(1)+(-1)(4) \end{pmatrix} \\ &= \begin{pmatrix} 28 & 17 \\ -2 & -2 \end{pmatrix} \end{aligned}$$

Therefore

$$AB = \begin{pmatrix} 28 & 17 \\ -2 & -2 \end{pmatrix}.$$

The Identity Matrix

The square matrix $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$, called the **identity matrix**, is a special matrix associated with all 2×2 matrices. It is denoted by the capital letter I.

If $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ is any 2×2 matrix, then

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \quad \text{and} \quad \begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}.$$

This gives us a very important result for matrices,

$$IA = A = AI.$$

Related Reading

Morris, O.D., P. Cooke. 1993. *Text & Tests 5.* The Celtic Press.