

## Solution Sets

### Aim

To introduce and explain the possible solution sets associated with systems of linear equations.

### Learning Outcomes

At the end of this section you will be able to:

- Tell the difference between a consistent and an inconsistent system of equations,
- Understand the different solution sets.

Not all systems of linear equations have solutions. Take for example the following system,

$$\begin{aligned}x + y &= 4 \\2x + 2y &= 6\end{aligned}$$

If we multiply the second equation of the system by  $\frac{1}{2}$  it is clear to see that there is no solution since the resulting system

$$\begin{aligned}x + y &= 4 \\x + y &= 3\end{aligned}$$

has contradictory equations.

A system of equations that has no solution is said to be inconsistent; if there is at least one solution of the system, it is said to be consistent. To demonstrate the possibilities that can occur in solving systems of linear equations, consider a general system of two linear equations in the unknowns  $x$  and  $y$ :

$$\begin{aligned}a_1x + b_1y &= c_1 && (a_1, b_1 \text{ not both zero}) \\a_2x + b_2y &= c_2 && (a_2, b_2 \text{ not both zero})\end{aligned}$$

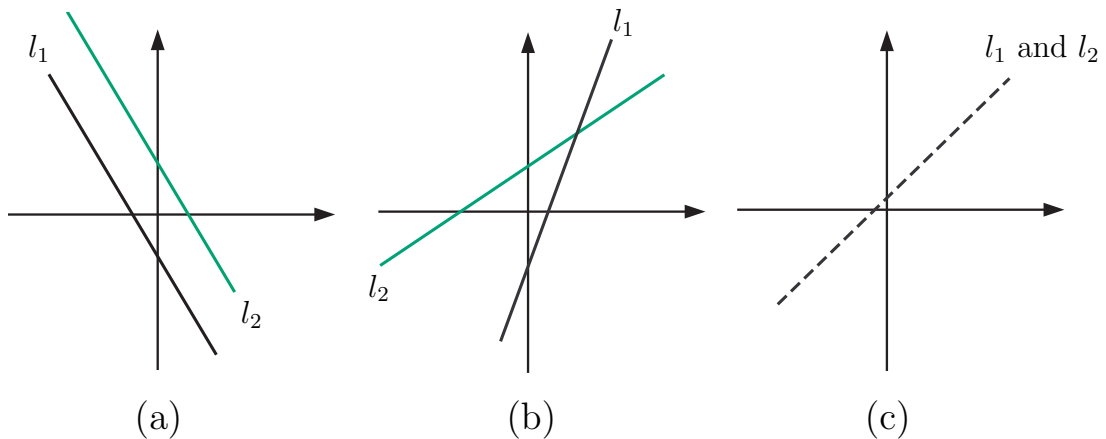
The graphs of these equations are lines, call them  $l_1$  and  $l_2$ . Since a point  $(x, y)$  lies on a line if and only if the numbers  $x$  and  $y$  satisfy the equation of the line, the solutions of the system of equations correspond to points of intersection of  $l_1$  and  $l_2$ .

There are three possibilities (show in the figure below):

- The lines  $l_1$  and  $l_2$  may be parallel, in which case there is no intersection and consequently no solution of the system.
- The lines  $l_1$  and  $l_2$  may intersect at only one point, in which case the system has exactly one solution.
- The lines  $l_1$  and  $l_2$  may coincide, in which case there are infinitely many points of intersection and consequently infinitely many solutions to the system.

These are the only possibilities when dealing with linear systems.

**Note:** Every system of linear equations has either no solutions, exactly one solution, or infinitely many solutions.



- (a) - No Solution
- (b) - One Solution
- (c) - Infinitely Many Solutions

## Related Reading

Anton, H. 1994. *Elementary Linear Algebra*. 7<sup>th</sup> Edition. John Wiley & Sons Inc.