# Trigonometric Functions

**Aim**

To introduce the trigonometric ratios sine, cosine and tangent.

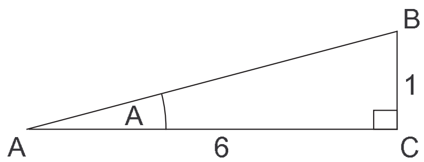
**Learning Outcomes**

At the end of this section you will be able to:

- Define the ratios sine, cosine and tangent with reference to a right angled triangle,
- Use the trigonometric ratios to solve problems involving triangles.

## The Tangent Ratio

The diagram below shows part of a steep road $AB$.




The steepness or *slope* of the road is measured by dividing the vertical distance by the horizontal distance, that is, $\frac{1}{6}$.

This slope is got by dividing the length of the side opposite the angle $A$ by the length of the side adjacent to $A$.

This ratio, $\frac{\text{opposite side}}{\text{adjacent side}}$, is called the **tangent** of the angle $A$ which we abbreviate to $\tan A$.

$$\tan A = \frac{opposite}{adjacent}$$

## The Sine Ratio

In the same right angled triangle as above, the ratio $\frac{\text{opposite side}}{\text{hypotenuse}}$ is called the **sine** of the angle $A$ which we abbreviate to $\sin A$.

$$\sin A = \frac{opposite}{hypotenuse}$$

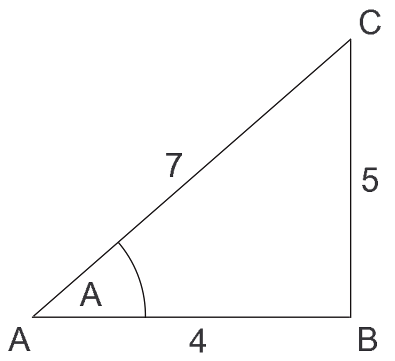
## The Cosine Ratio

In the right angled triangle shown on page 1, the ratio $\frac{\text{adjacent side}}{\text{hypotenuse}}$ is called the **cosine** of the angle $A$ which we abbreviate to $\cos A$.

$$\cos A = \frac{adjacent}{hypotenuse}$$

<u>Example 1</u>

Calculate sin, cos and tan of the angle $A$ in the following triangle.




$$\sin A = \frac{BC}{AC} = \frac{5}{7}\,, \qquad \cos A = \frac{AB}{AC} = \frac{4}{7}\,, \qquad \tan A = \frac{BC}{AB} = \frac{5}{4}.$$

Note: Since the hypotenuse is always the longest side of a right angled triangle, sin and cos can never be greater than 1. Can you see why ?

## Using a Calculator

All of these ratios have already been worked out and are available in published tables. Another way of working out the ratios is to use a scientific calculator. Recall that an angle may be measured in degrees or in radians. An angle in degrees has the symbol $^\circ$, otherwise assume that the angle is in radians. Remember that it may be necessary to switch your calculator from degrees mode to radians mode or vice versa. Study your calculator manual to learn how to do this.

<u>Example 2</u>

Check that $\sin 48^\circ = 0.743145$ and $\cos 1.4$ radians $= 0.169967$.

## Reversing the Process

If we are given a value for $\sin A$, $\cos A$ or $\tan A$ we may want to work out the corresponding angle $A$. This process is known as finding the inverse. Your calculator can assist you in doing this. There may be an INV(shift) key on your calculator, or a $\sin^{-1}$ key. Similar keys will exist for $\cos^{-1}$ and $\tan^{-1}$. Check that you can use your calculator to show that if $\cos A = 0.75$ then $A \approx 41°24'$ or 0.722734... radians.

## Related Reading

Booth, D.J. 1998. *Foundation Mathematics.* $3^{rd}$ Edition. Pearson Education Limited.

Croft, A., R. Davison. 2003. *Foundation Mathematics.* $3^{rd}$ Edition. Pearson Education Limited.