What is a Vector ?

Aim

To define, in simple terms, what a vector is.

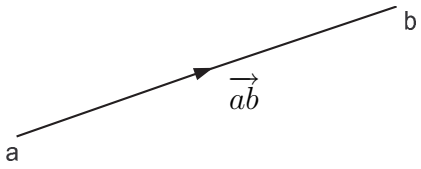
Learning Outcomes

At the end of this section you will be able to:

- Understand what a vector is,
- Tell the difference between a free vector and a position vector,
- Understand the basic concepts of vectors.

Many physical quantities such as area, length, mass and temperature are fully described once the magnitude(measure) of the quantity is given. Such quantities are called **scalars**. Other physical quantities, called **vectors**, are not completely determined until both a magnitude(length) and a direction are specified. For example, winds are usually described by their speed and direction. The wind speed and the wind direction together form a separate vector called the wind velocity. Other example of vectors are force and displacement.

The displacement from a to b in the figure below is called the vector ab and can be represented by $\overrightarrow{ab}$. (It is also common practice to represent vectors using bold notation, i.e $\overrightarrow{u} = \mathbf{u}$.)



If the arrow on the vector was facing in the opposite direction, i.e. pointing from b to a, then this would then become the vector $\overrightarrow{ba}$.

Equal Vectors

Two vectors are equal if they have the same magnitude (length) and direction. From a graphical point of view, the two vectors will be of equal length, parallel (same direction) and the arrows will be pointing in the same direction.

The Magnitude of a Vector

The *magnitude* of a vector is the length of the line segment that represents the vector. The modulus of a vector $\overrightarrow{a}$ is represented by $|\overrightarrow{a}|$. If the 3 dimensional vector $\overrightarrow{a}$ was written in *component form*, i.e. $\overrightarrow{a} = \langle a_1, a_2, a_3 \rangle$, then the magnitude of $\overrightarrow{a}$ would be given by

$$|\overrightarrow{a}| = \sqrt{a_1^2 + a_2^2 + a_3^2}.$$

Example

Find $|\overrightarrow{a}|$ if $\overrightarrow{a} = \langle 2, 3, -1 \rangle$.

Solution

$|\overrightarrow{a}| = \sqrt{2^2 + 3^2 + (-1)^2} = \sqrt{14}$.

The Negative of a Vector

If two vectors, $\overrightarrow{a}$ and $\overrightarrow{b}$ with the same magnitude(length) are parallel, and are only different in the direction that the arrows of the vectors point, then vector $\overrightarrow{b}$ is said to be the negative of vector $\overrightarrow{a}$ and visa versa, i.e. $\overrightarrow{b} = -\overrightarrow{a}$.

Free Vectors and Position Vectors

A *free* vector is a vector that is free to change its location whilst still representing the same vector. If a vector $\overrightarrow{v}$ is moved under parallel displacement from its original location then the resulting vector will still be $\overrightarrow{v}$ because both the magnitude and direction of the vector have remained the same.

A *position* vector on the other hand, is a vector which has its starting point fixed. For this reason if a position vector was moved under parallel displacement the resulting vector, regardless of whether it maintains the magnitude and direction of the original vector, would be different from the original vector because its starting location will have changed.

Properties of Vectors

If $\overrightarrow{u}$, $\overrightarrow{v}$ and $\overrightarrow{w}$ are vectors in 2 or 3 dimensional space and a and b are scalars, then

- $\overrightarrow{u} + \overrightarrow{v} = \overrightarrow{v} + \overrightarrow{u}$
- $\overrightarrow{u} + (\overrightarrow{v} + \overrightarrow{w}) = (\overrightarrow{u} + \overrightarrow{v}) + \overrightarrow{w}$

- $\overrightarrow{u} + \overrightarrow{0} = \overrightarrow{u}$
- $\overrightarrow{u} + (-\overrightarrow{u}) = \overrightarrow{0}$
- $a(\overrightarrow{u} + \overrightarrow{v}) = a\overrightarrow{u} + a\overrightarrow{v}$
- $(a+b)\overrightarrow{u} = a\overrightarrow{u} + b\overrightarrow{u}$
- $(ab)\overrightarrow{u} = a(b\overrightarrow{u})$
- $1\overrightarrow{u} = \overrightarrow{u}$

Related Reading

Adams, R.A. 2003. *Calculus: A Complete Course.* 5^{th} Edition. Pearson Education Limited.

Anton, H., I. Bivens, S. Davis. 2005. *Calculus.* 8^{th} Edition. John Wiley & Sons.

Morris, O.D., P. Cooke. 1993. *Text & Tests 5.* The Celtic Press.