Inequalities involving the Absolute Value Symbol

Aim

To demonstrate how to solve inequalities involving the absolute value symbol.

Learning Outcomes

At the end of this section you will be able to:

- Find the absolute value of any real number,
- Solve inequalities involving the absolute value symbol.

Absolute Value

The absolute value of x, denoted $|x|$ (and which is read as "the absolute value of x"), can be thought of as the distance from x to zero. For this reason the absolute value is never negative. The absolute value symbol, $|\ |$, is sometimes referred to as the *modulus* symbol.

The absolute value can be officially defined as

$$|x| = \sqrt{x^2}.$$

Example 1

Find the absolute value of (i) 6 and (ii) -8.

(i) $|6| = \sqrt{6^2} = \sqrt{36} = 6,$ (ii) $|-8| = \sqrt{(-8)^2} = \sqrt{64} = 8.$

From this it can be seen that the absolute value is **always** positive.

Absolute Values and Inequalities

The absolute value value sign is often used in conjunction with inequalities. For example, $|x| > 3$ means all the real numbers x whose distance from the origin is greater than 3 units. This is clearly referring to the interval $(-\infty, -3) \cup (3, +\infty)$. Thus

$$|x| > 3 \qquad \text{means} \qquad x < -3 \quad \text{or} \quad x > 3.$$

Based on this result it is now possible to solve inequalities involving absolute values.

Example 2

Solve the inequality, $|x-6|>2$.

This is equivalent to

$$x-6<-2 \quad \text{or} \quad x-6>2,$$

$$\Rightarrow x-6+6<-2+6 \quad \text{or} \quad x-6+6>2+6,$$

$$\Rightarrow x<4 \quad \text{or} \quad x>8.$$

Example 3

Solve the inequality, $|2x+3|<6$.

This is equivalent to

$$\begin{aligned}
-6 &< 2x+3 &&< 6, && \text{subtract 3 from everything,}\\
-6-3 &< 2x+3-3 &&< 6-3, &&\\
-9 &< 2x &&< 3, && \text{divide everything by 2,}\\
-\frac{9}{2} &< x &&< \frac{3}{2}. &&
\end{aligned}$$

Related Reading

Morris, O.D. 1987. *Text & Tests 1.* The Celtic Press.

Booth, D.J. 1998. *Foundation Mathematics.* 3^{rd} Edition. Pearson Education Limited.