Applications of Differentiation

Aim

To demonstrate some real world applications of differentiation.

Learning Outcomes

At the end of this section you will be able to:

- Identify several areas of application for differentiation,
- Apply your knowledge of differentiation to solve real world problems.

There are many applications of differentiation in the area of *rates of change*. Some of the areas examined in this course are velocity and acceleration and volumes and areas.

Velocity and Acceleration

In the study of motion in a straight line, equations of the form

$$s = at^2 + bt + c,$$

where s is the distance, t is the time and a, b, c are constants frequently occur. A change in t results in a change in the value of s. The rate of change of s with respect to t is given by $\dfrac{ds}{dt}$,

$$\text{i.e.} \quad \frac{ds}{dt} = \text{velocity } (v).$$

The rate of change of velocity (or speed) with respect to time is called the acceleration, a, of the object and is given by,

$$\text{acceleration } (a) = \frac{dv}{dt} = \frac{d^2s}{dt^2}.$$

Based on these formulas it is now possible to solve problems based on the velocity and acceleration of objects moving in a straight line using differentiation.

Example

A particle is moving in a straight line and its distance s, in metres, from a fixed point in the line after t seconds is given by the equation, $s = 12t - 15t^2 + 4t^3$. Find

1. the velocity and acceleration of the particle after 3 seconds,
2. the distance travelled between the two points when the velocity is instantaneously zero.

Part (1).

$$\begin{aligned} s &= 12t - 15t^2 + 4t^3, \\ \Rightarrow \frac{ds}{dt} &= 12 - 30t + 12t^2, \end{aligned}$$

at $t = 3$ this becomes

$$12 - 30(3) + 12(3)^2 = 30.$$

Therefore the velocity is 30m/sec after 3 seconds.

$$\frac{d^2s}{st^2} = -30 + 24t,$$
$$\Rightarrow -30 + 24(3) = 42.$$

Therefore the acceleration is 42m/sec^2 after 3 seconds.

Part (2).

Velocity is zero when $\frac{ds}{dt} = 0$.

$$\begin{aligned} 12 - 30t + 12t^2 &= 0, \\ 2t^2 - 5t + 2 &= 0, \\ (2t - 1)(t - 2) &= 0. \end{aligned}$$

Therefore the velocity is zero when $t = \frac{1}{2}$ and $t = 2$.

When $t = \frac{1}{2} \Rightarrow s = 12(\frac{1}{2}) - 15(\frac{1}{2})^2 + 4(\frac{1}{2})^3 = 2\frac{3}{4}$ metres,
When $t = 2 \Rightarrow s = 12(2) - 15(2)^2 + 4(2)^3 = -4$ metres.

Therefore the distance travelled between $t = \frac{1}{2}$ and $t = 2$ is

$$2\frac{3}{4} + |-4| = 6\frac{3}{4} \text{ metres.}$$

As mentioned earlier there are several other practical application of the use of differentiation as a measure of the rate of change of a function with respect to a variable. For example, if $A = \pi r^2$, where A is the area of a circle and r is the radius then $\frac{dA}{dr}$ is the rate of change of area with respect to r, the radius. Normally however, most rates of change are measured with respect to time. For example, if V represents the volume, then $\frac{dV}{dt}$ is the rate of change of volume with respect to time.

Related Reading

Stroud, K.A. 2001. *Engineering Mathematics.* 5^{th} Edition. PALGRAVE.

Adams, R.A. 2003. *Calculus: A Complete Course.* 5^{th} Edition. Pearson Education Limited.

Morris, O.D., P. Cooke. 1992. *Text & Tests 4.* The Celtic Press.