Applications of Differentiation

Aim
To demonstrate some real world applications of differentiation.

Learning Outcomes
At the end of this section you will be able to:

- Identify several areas of application for differentiation,
- Apply your knowledge of differentiation to solve real world problems.

There are many applications of differentiation in the area of rates of change. Some of the areas examined in this course are velocity and acceleration and volumes and areas.

Velocity and Acceleration
In the study of motion in a straight line, equations of the form

\[ s = at^2 + bt + c, \]

where \( s \) is the distance, \( t \) is the time and \( a, b, c \) are constants frequently occur. A change in \( t \) results in a change in the value of \( s \). The rate of change of \( s \) with respect to \( t \) is given by \( \frac{ds}{dt} \).

i.e. \( \frac{ds}{dt} = \) velocity \( (v) \).

The rate of change of velocity (or speed) with respect to time is called the acceleration, \( a \), of the object and is given by,

\[ \text{acceleration (a)} = \frac{dv}{dt} = \frac{d^2s}{dt^2}. \]

Based on these formulas it is now possible to solve problems based on the velocity and acceleration of objects moving in a straight line using differentiation.

Example
A particle is moving in a straight line and its distance \( s \), in metres, from a fixed point in the line after \( t \) seconds is given by the equation, \( s = 12t - 15t^2 + 4t^3 \). Find
1. the velocity and acceleration of the particle after 3 seconds,

2. the distance travelled between the two points when the velocity is instantaneously zero.

Part (1).

\[ s = 12t - 15t^2 + 4t^3, \]
\[ \Rightarrow \frac{ds}{dt} = 12 - 30t + 12t^2, \]

at \( t = 3 \) this becomes

\[ 12 - 30(3) + 12(3)^2 = 30. \]

Therefore the velocity is 30m/sec after 3 seconds.

\[ \frac{d^2s}{st^2} = -30 + 24t, \]
\[ \Rightarrow -30 + 24(3) = 42. \]

Therefore the acceleration is 42m/sec^2 after 3 seconds.

Part (2).

Velocity is zero when \( \frac{ds}{dt} = 0. \)

\[ 12 - 30t + 12t^2 = 0, \]
\[ 2t^2 - 5t + 2 = 0, \]
\[ (2t - 1)(t - 2) = 0. \]

Therefore the velocity is zero when \( t = \frac{1}{2} \) and \( t = 2. \)

When \( t = \frac{1}{2} \) \( \Rightarrow s = 12\left(\frac{1}{2}\right) - 15\left(\frac{1}{2}\right)^2 + 4\left(\frac{1}{2}\right)^3 = 2\frac{3}{4} \) metres,

When \( t = 2 \) \( \Rightarrow s = 12(2) - 15(2)^2 + 4(2)^3 = -4 \) metres.

Therefore the distance travelled between \( t = \frac{1}{2} \) and \( t = 2 \) is

\[ \frac{3}{4} + | -4| = 6\frac{3}{4} \] metres.
The Derivative
and its Applications

As mentioned earlier there are several other practical application of the use of differ-
entiation as a measure of the rate of change of a function with respect to a variable.
For example, if $A = \pi r^2$, where $A$ is the area of a circle and $r$ is the radius then $\frac{dA}{dr}$ is
the rate of change of area with respect to $r$, the radius. Normally however, most rates
of change are measured with respect to time. For example, if $V$ represents the volume,
then $\frac{dV}{dt}$ is the rate of change of volume with respect to time.

Related Reading


Limited.