

Applications of Trigonometry

Aim

To demonstrate how trigonometry can be used to solve real world problems.

Learning Outcomes

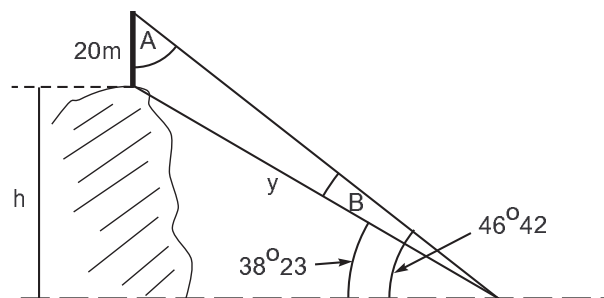
At the end of this section you will be able to:

- See how trigonometry can be used to solve real world problems,
- Apply trigonometry to help in solving real world problems.

The earliest applications of trigonometry were in the fields of navigation, surveying, and astronomy, in which the main problem generally was to determine an inaccessible distance, such as the distance between the earth and the moon, or of a distance that could not be measured directly, such as the distance across a large lake. Over the years many more applications of trigonometry have been found. The applications mentioned in most textbooks and courses on trigonometry are generally navigation, surveying, and construction based.

Example

A 20m high mast is placed on top of a cliff whose height above sea level is unknown. An observer as sea level sees the top of the mast at an elevation of $46^{\circ}42'$ and the foot of the mast at $38^{\circ}23'$. Find h , the height of the cliff.



The angle A can easily be computed seeing as it is the missing angle in a right-angled triangle with $46^{\circ}42'$ forming the other angle. Therefore

$$A = 90^{\circ} - 46^{\circ}42' = 43^{\circ}18'.$$

Similarly the angle B can easily be computed by subtracting the angle to the foot of the mast from the angle to the top of the mast, i.e.

$$B = 46^\circ 42' - 38^\circ 23' = 8^\circ 19'.$$

Using the sine rule it can be seen that

$$\frac{h}{\sin 38^\circ 23'} = \frac{y}{\sin 90^\circ}.$$

Since $\sin 90^\circ = 1$, we have that

$$h = y \sin 38^\circ 23'. \quad (1)$$

The next step is to find the length of y . Using the small (non-right angled) triangle consisting of the angles A and B and the sine rule we have that

$$\frac{y}{\sin A} = \frac{20}{\sin B}.$$

From this we then get

$$y = 20 \frac{\sin A}{\sin B},$$

and by filling in for the angles A and B this becomes

$$y = 20 \frac{\sin 43^\circ 18'}{\sin 8^\circ 19'}.$$

Using this result in equation (1) gives

$$\begin{aligned} h &= 20 \frac{\sin 43^\circ 18'}{\sin 8^\circ 19'} \sin 38^\circ 23', \\ &= 20 \frac{\sin 43^\circ 18' \sin 38^\circ 23'}{\sin 8^\circ 19'}, \\ &\approx 59m. \end{aligned}$$

Related Reading

Booth, D.J. 1998. *Foundation Mathematics*. 3rd Edition. Pearson Education Limited.