

Area of a Triangle

Aim

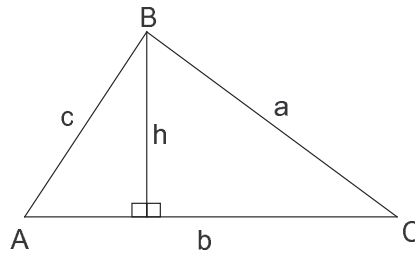
To demonstrate how to calculate the area of a triangle using trigonometry.

Learning Outcomes

At the end of this section you will be able to:

- Derive the formula for calculating the area of a triangle using trigonometry,
- Calculate the area of a triangle using trigonometry.

We are all familiar with the formula for the area of a triangle, $\frac{1}{2}bh$, where b stands for the length of the base and h stands for the perpendicular height. In the triangle below the area can be expressed as $\frac{1}{2}bh$.



Now by using the right angled triangle on the left side of the diagram, and our knowledge of trigonometry, we can state that:

$$\sin A = \frac{h}{c},$$

$$c \sin A = h.$$

This tells us that the height, h , can be expressed as $c \sin A$. If we substitute this new expression for the height, we can write the area formula as:

$$\frac{1}{2}bc \sin A.$$

The letters of the above formula can be swapped around to give formula for the area of a triangle. In general, the area of a triangle is given by

$$\frac{1}{2}bc \sin A \quad \text{or} \quad \frac{1}{2}ac \sin B \quad \text{or} \quad \frac{1}{2}ab \sin C.$$

To use this formula you must be given the value of two sides and the angle between them.

Example

Calculate the area of the triangle ABC , to 2 decimal places, using the following information $AB = 7\text{cm}$, $BC = 6\text{cm}$, $AC = 5\text{cm}$, $B = 50^\circ$.

Since we are given the angle B , we need to find the two sides that meet at the vertex B . The two sides are clearly AB and BC . We also know that $AB \equiv c$ and $BC \equiv a$. It is now possible to calculate the area.

$$\begin{aligned}\text{Area} &= \frac{1}{2}ac \sin B, \\ &= \frac{1}{2}(6)(7) \sin(50^\circ), \\ &= \frac{1}{2} \times 42 \times (0.766044443), \\ &= 16.08693331\end{aligned}$$

Therefore,

$$\text{Area} \approx 16.09\text{cm}^2.$$

Related Reading

Morris, O.D., P. Cooke. 1992. *Text & Tests 4*. The Celtic Press.