Linear Inequalities

Aim

To demonstrate how to solve linear inequalities.

Learning Outcomes

At the end of this section you will be able to:

- Identify the difference between an inequality and an equation,
- Manipulate linear inequalities,
- Solve linear inequalities.

An inequality is an expression which includes at least one of the following mathematical symbols ($<$, $>$, $\leq$, $\geq$). **Linear inequalities** are sometimes referred to as *basic inequalities*. In this section we will learn how to solve these "basic" inequalities. A "*solution*" of an inequality is a number which when substituted for the variable makes the inequality a true statement.

Solving linear inequalities is very similar to solving linear equations. For example,

$$\begin{aligned} x+4 &> 5, \qquad \text{we subtract 4 from both sides to isolate } x, \\ \Rightarrow (x+4)-4 &> 5-4, \\ \Rightarrow x &> 1. \end{aligned}$$

The only difference here is that you have a "greater than" sign, instead of an "equals" sign.

A major difficulty in solving inequalities occurs when it is necessary to multiply or divide the expression by a negative number. In these cases it is also necessary to **reverse the inequality sign**.

For example,

$$\begin{aligned} 5-x &\leq 6, \qquad \text{we subtract 5 from both sides to isolate } x, \\ \Rightarrow (5-x)-5 &\leq 6-5, \\ \Rightarrow -x &\leq 1, \qquad \text{now multiply by } -1 \text{ to solve for } x, \\ \Rightarrow x &\geq -1. \end{aligned}$$

Rules for Manipulating Inequalities

When changing or rearranging statements involving inequalities the following rules should be followed:

- **Rule 1**: Adding or subtracting the same quantity from both sides of an inequality leaves the inequality symbol unchanged.
- **Rule 2**: Multiplying or dividing by a **positive** number leaves the inequality symbol unchanged.
- **Rule 3**: Multiplying or dividing by a **negative** number **reverses the inequality sign**. For example, $>$ changes to $<$, and vice versa.
- **Rule 4**: Inverting fractions **reverses the inequality sign**. For example, $\frac{2}{5} < \frac{2}{3} \Rightarrow \frac{5}{2} > \frac{3}{2}$.

It is also possible to solve inequalities that consist of both algebraic and numeric terms on both sides of the inequality sign. In this case it is necessary to group all the algebraic terms together on one side and group all the numeric terms together on the other side of the inequality and then simplify using the rules above. For example,

$$\begin{aligned}
2(x-4) &> 3(2x+4), && \text{multiply out brackets,} \\
\Rightarrow 2x-8 &> 6x+12, && \\
\Rightarrow (2x-8)-2x &> (6x+12)-2x, && \text{subtract } 2x \text{ from both sides,} \\
\Rightarrow -8 &> 4x+12, && \\
\Rightarrow -8-12 &> (4x+12)-12, && \text{subtract 12 from both sides,} \\
\Rightarrow -20 &> 4x, && \text{finally divide both sides by 4,} \\
\Rightarrow -5 &> x. &&
\end{aligned}$$

Related Reading

Morris, O.D. 1987. *Text & Tests 1.* The Celtic Press.

Booth, D.J. 1998. *Foundation Mathematics.* 3^{rd} Edition. Pearson Education Limited.