

Degrees and Radians

Aim

To explain the units *degrees* and *radians*.

Learning Outcomes

At the end of this section you will be able to:

- Distinguish between degrees and radians,
- Convert from degrees to radians and vice versa.

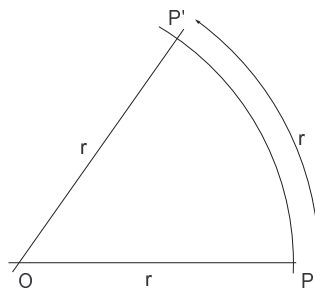
A triangle is a three-sided figure. The point where any two sides meet is called a **vertex** of the triangle and at each vertex is an **angle**. An angle is a measure of rotation. We use angles to measure the amount by which a line has been turned. An angle can be measured in units of degrees or radians.

Degrees

A full revolution is 360 degrees, denoted 360° . Each degree is subdivided into 60 minutes ($'$) and each minute further subdivided into 60 seconds ($''$). 180° is sometimes called a straight line angle and 90° is called a right angle.

Radians

The radian is an alternative unit of measure of an angle, the use of which is imperative in certain situations that we shall meet later. When OP is rotated about O both P and P' lie on the circumference of a circle with radius $r = OP$. If the length of the arc PP' is equal to the radius OP then the line OP' has rotated through an angle defined as 1 radian.



Since one complete revolution causes the point P' to move the full circumference $2\pi r$ then one complete revolution is equal to 2π radians.

Converting Degrees to Radians

Angles measured in degrees can be converted to angles measured in radians and vice versa:

$$360 \text{ degrees} = 2\pi \text{ radians}$$

and therefore

$$\begin{aligned} 1 \text{ degree} &= \frac{2\pi}{360} \text{ radians} \\ &= \frac{\pi}{180} \text{ radians} && \pi \approx 3.14159 \\ &= 0.017453\dots \text{radians} \end{aligned}$$

and

$$\begin{aligned} 1 \text{ radian} &= \frac{360}{2\pi} \text{ degrees} \\ &= \frac{180}{\pi} \text{ degrees} && \pi \approx 3.14159 \\ &= 57.2957\dots \text{degrees} \end{aligned}$$

In general,

$$x^\circ = x \times \frac{\pi}{180} \text{ radians.}$$

Related Reading

Booth, D.J. 1998. *Foundation Mathematics*. 3rd Edition. Pearson Education Limited.

Croft, A., R. Davison. 2003. *Foundation Mathematics*. 3rd Edition. Pearson Education Limited.