Euler's Formula

Aim

To explain what Euler's Formula is and to demonstrate its usage.

Learning Outcomes

At the end of this section you will be able to:

- Write a complex number in exponential polar form,
- Simplify complex numbers written using Euler's formula.

Euler's Formula which is presented on page 9 of the log tables states that

$$e^{i\theta} = (\cos\theta + i\sin\theta).$$

From this it is easy to see that a complex number, when written in polar form, i.e

$$z = r(\cos\theta + i\sin\theta)$$

can easily be converted in exponential polar form using Euler's formula to give

$$z = re^{i\theta}.$$

Example

Simplify the following complex number $z = e^{(2+i\frac{\pi}{2})}$.

We know that $a^{n+m} = a^n \times a^m$, therefore

$$\begin{aligned} e^{(2+i\frac{\pi}{2})} &= e^2.e^{i\frac{\pi}{2}}, \\ &= e^2 \times \left(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right), \quad \text{using Euler}'\text{s Formula}, \\ &= e^2(0+i) = ie^2. \end{aligned}$$

Related Reading

Morris, O.D., P. Cooke. 1992. *Text & Tests 4.* The Celtic Press.

Stroud, K.A. 2001. *Engineering Mathematics.* 5^{th} Edition. PALGRAVE.