

Intervals

Aim

To introduce the concept of an interval and explain the difference between an open and a closed interval.

Learning Outcomes

At the end of this section you will be able to:

- Differentiate between an open and a closed interval,
- Determine the elements of any given interval.

An interval is a connected portion of the real line. If the endpoints a and b are finite and are included, the interval is called **closed** and is denoted $[a, b]$.

If the endpoints are not included, the interval is called **open** and denoted (a, b) .

If one endpoint is included but not the other, the interval is denoted $[a, b)$ or $(a, b]$ and is called a half-closed (or half-open interval, depending on your preference).

Specifically we define for the real numbers a and b :

- $[a, b]$: the set of all numbers x satisfying $a \leq x \leq b$,
- (a, b) : the set of all numbers x satisfying $a < x < b$,
- $[a, b)$: the set of all numbers x satisfying $a \leq x < b$,
- $(a, b]$: the set of all numbers x satisfying $a < x \leq b$.

Another case worth considering is if one of the endpoints is $\pm\infty$. This means that the interval is unlimited on the right or left. For example, the interval $(1, \infty)$ is just another way of describing all the number greater than 1, and $(-\infty, 10]$ is the set of all numbers less than or equal to 10.

The set of all real numbers can be expressed as $(-\infty, \infty)$.

If the set consists of several disconnected pieces, we use the symbol \cup (union) to join the different pieces.

For example,

$$[-5, -2] \cup (2, \infty)$$

can be used to represent all the numbers between -5 and -2 inclusively, and all the numbers greater than 2.

Related Reading

Morris, O.D. 1987. *Text & Tests 1*. The Celtic Press.