

Multiplying and Dividing Complex Numbers

Aim

To demonstrate how to Multiply and Divide complex numbers.

Learning Outcomes

At the end of this section you will be able to:

- Multiply two complex numbers,
- Divide two complex numbers.

Multiplication of Complex Numbers

The multiplication of complex numbers obeys the same laws as multiplication of algebraic binomials, with the exception that $i^2 = -1$.

Example 1

$$\begin{aligned}(2 - 3i)(4 + 2i) &= 8 + 4i - 12i - 6i^2, & (i^2 = -1), \\ &= 8 + 4i - 12i + 6, \\ &= 14 - 8i.\end{aligned}$$

Division of Complex Numbers

Division of complex numbers such as $\frac{2 + 3i}{3 - 4i}$ is performed by multiplying the numerator and the denominator by the conjugate of the denominator. This results in the denominator becoming a real number as shown in the next example.

Example 2

$$\begin{aligned}\frac{2 + 3i}{3 - 4i} &= \frac{2 + 3i}{3 - 4i} \times \frac{3 + 4i}{3 + 4i}, \\ &= \frac{(2 + 3i)(3 + 4i)}{(3 - 4i)(3 + 4i)}, \\ &= \frac{6 + 8i + 9i + 12i^2}{9 + 12i - 12i - 16i^2}, \\ &= \frac{6 - 12 + 17i}{9 + 16} = \frac{-6 + 17i}{25},\end{aligned}$$

$$\Rightarrow \frac{2 + 3i}{3 - 4i} = \frac{-6}{25} + \frac{17}{25}i.$$

Note: If z_1 and z_2 are two unrelated (not the conjugate of each other) complex numbers then the result of $z_1 \times z_2$ will always be a complex number. If, on the other hand, you multiply z_1 by its conjugate the result is a real number, i.e. $z_1 \times \bar{z}_1 = \text{real number}$.

Related Reading

Morris, O.D., P. Cooke. 1992. *Text & Tests 4*. The Celtic Press.

Stroud, K.A. 2001. *Engineering Mathematics*. 5th Edition. PALGRAVE.