Number Systems

Aim

To review the basics of number systems and introduce the student to the different types of number systems that they will encounter.

Learning Outcomes

At the end of this section you will be able to:

- Identify the different number systems,
- Identify the elements of the different number systems,
- Distinguish between the different number systems.

Set Theory

Before introducing the various number systems, a brief summary of **set theory** is provided. In mathematics a **set** is simply a collection of objects. The objects in a set are called the **elements** of the set. When it is possible to list the elements of a set, we write down the list inside a **pair of chain brackets** and separate each element from the next with a comma. We generally use **capital letters** to name or denote sets.

If A = the set of whole numbers greater than 4 and less than 10, then

$$A = \{5, 6, 7, 8, 9\}.$$

We use the symbol $\in$ to denote *is an element of*. In the set $A = \{5, 6, 7, 8, 9\}$,

$$5 \in A, \quad 6 \in A, \quad 7 \in A, \quad 8 \in A \quad \text{and} \quad 9 \in A.$$

We use the symbol $\notin$ to denote *is not an element of*. In the above example, $10 \notin A$.

Sometimes when a set contains a large number of elements it is convenient to describe the elements of the set rather than to list them all. For this purpose we use a **rule method** to describe the set.

If $A = \{\text{Spring, Summer, Autumn, Winter}\}$, then the set A can be described as

$$A = \{x | x \text{ is a season of the year}\}.$$

The notation reads as follows:

A equals the set of all elements x such that x is a season of the year.

The **null set** or **empty set** is represented by $\emptyset$ or $\{\ \}$.

Sometimes we find that every element of one set is also an element of some other set. When this situation exists, one set is said to be a **subset** of the other set. We use the notation $\subset$ to indicate *is a subset of.*

If

$$A = \{3, 4, 5, 6, 7\} \quad \text{and} \quad B = \{5, 7\},$$

we notice that all the elements of B are contained in A and so B is said to be a subset of A,

$$B \subset A.$$

Natural Numbers - $\mathbb{N}$

In mathematics we generally refer to the counting numbers as the set of **natural numbers**. Whether or not zero should be included or omitted from the set of natural numbers is debateable. We use the capital letter $\mathbb{N}$ to denote this set.

$$\mathbb{N} = \{1, 2, 3, 4, 5, ...\}.$$

There is no last number to the set $\mathbb{N}$. We call it an **infinite set**.

Integers - $\mathbb{Z}$

The set of positive and negative whole numbers is called the **set of integers**. It is denoted by the capital letter $\mathbb{Z}$.

$$\mathbb{Z} = \{..., -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, ...\}.$$

Rational Numbers - $\mathbb{Q}$

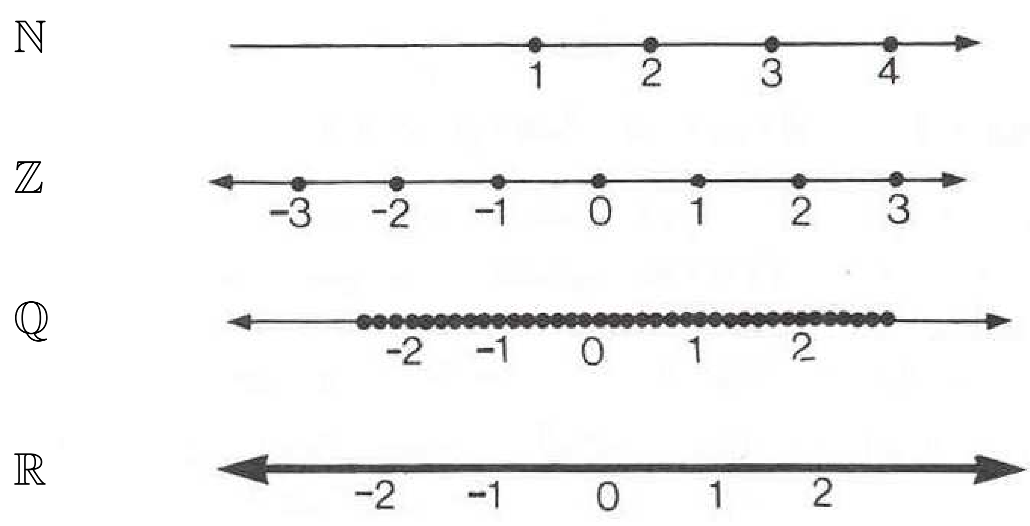
A **rational number** is defined as any number that can be written in the form $\frac{a}{b}$ where $b \neq 0$. Thus, all fractions are rational numbers. The set of rational numbers is denoted by the letter $\mathbb{Q}$. Fractions which are greater than 1 are also rational numbers as they can be written in the form $\frac{a}{b}$. Fractions such as these are called **improper fractions** as they are greater than 1, e.g. $\frac{5}{3}$.

Real Numbers - $\mathbb{R}$

Some decimals neither terminate nor repeat. For example, $\sqrt{2} = 1.414213562...$ Decimals such as these are known as **irrational numbers**. An irrational number cannot be expressed in the form $\frac{a}{b}$, where a and b are integers.

When the set of rational numbers is combined with the set of irrational numbers they form the set of **real numbers**. Any number on the numberline is a real number. We use the letter $\mathbb{R}$ to denote the set of real numbers.

The diagram below shows how natural numbers ($\mathbb{N}$), integers ($\mathbb{Z}$), rational numbers ($\mathbb{Q}$) and real numbers ($\mathbb{R}$) can be illustrated on the numberline.



Related Reading

Morris, O.D. 1987. *Text & Tests 1.* The Celtic Press.

Booth, D.J. 1998. *Foundation Mathematics.* 3^{rd} Edition. Pearson Education Limited.