

Proving Trigonometric Identities

Aim

To demonstrate how to prove trigonometric identities.

Learning Outcomes

At the end of this section you will be able to:

- Distinguish between an equation and an identity,
- Solve trigonometric identities.

An equation is an expression that is true for some value(s) of the unknown variable. An **identity** is an expression that is true for **all** values of the unknown variable. Some example of trigonometric identities are

$$\begin{aligned}\cos^2 \theta + \sin^2 \theta &= 1, \\ \tan \theta &= \frac{\sin \theta}{\cos \theta}.\end{aligned}$$

Page 9 of the log tables can be very useful when proving trigonometric identities. We'll now look at a two examples.

Example 1

$$\text{Prove } \cos^2 \theta = \frac{1 + \cos 2\theta}{2}.$$

From the log tables we know that

$$\begin{aligned}\cos(A + B) &= \cos A \cos B - \sin A \sin B \quad (\text{let } A = \theta \text{ and } B = \theta), \\ \Rightarrow \cos 2\theta &= \cos^2 \theta - \sin^2 \theta.\end{aligned}\tag{1}$$

Also,

$$\begin{aligned}\cos^2 \theta + \sin^2 \theta &= 1, \\ \Rightarrow \sin^2 \theta &= 1 - \cos^2 \theta.\end{aligned}\tag{2}$$

Substitute equation (2) into equation (1) and we get,

$$\begin{aligned}\cos 2\theta &= \cos^2 \theta - (1 - \cos^2 \theta), \\ &= \cos^2 \theta - 1 + \cos^2 \theta, \\ \Rightarrow \cos 2\theta &= 2 \cos^2 \theta - 1.\end{aligned}$$

$$\begin{aligned}2 \cos^2 \theta &= \cos 2\theta + 1, \\ \Rightarrow \cos^2 \theta &= \frac{1 + \cos 2\theta}{2}.\end{aligned}$$

Example 2

Prove that $\tan^2 \theta + 1 = \sec^2 \theta$.

Recall that $\sec \theta = \frac{1}{\cos \theta}$ and $\tan \theta = \frac{\sin \theta}{\cos \theta}$. Therefore,

$$\begin{aligned}\tan^2 \theta + 1 &= \sec^2 \theta, \\ \frac{\sin^2 \theta}{\cos^2 \theta} + 1 &= \frac{1}{\cos^2 \theta}, \\ \frac{\sin^2 \theta + \cos^2 \theta}{\cos^2 \theta} &= \frac{1}{\cos^2 \theta}, \\ \Rightarrow \frac{1}{\cos^2 \theta} &= \frac{1}{\cos^2 \theta}.\end{aligned}$$

Related Reading

Croft, A., R. Davison. 2003. *Foundation Mathematics*. 3rd Edition. Pearson Education Limited.

Booth, D.J. 1998. *Foundation Mathematics*. 3rd Edition. Pearson Education Limited.